

The Memory Hierarchy & Cache

- Review of Memory Hierarchy & Cache Basics (from 350):

Review
From
350

- Motivation for The Memory Hierarchy:

- CPU/Memory Performance Gap
- The Principle Of Locality

Cache \$\$\$\$\$

- Cache Basics:

- Block placement strategy & Cache Organization:
- Block replacement policy
- Unified vs. Separate Level 1 Cache

- CPU Performance Evaluation with Cache:

- Average Memory Access Time (AMAT)/Memory Stall cycles
- Memory Access Tree

Cache exploits memory access locality to:

- 1 Lower AMAT by hiding long main memory access latency.
Thus cache is considered a memory latency-hiding technique.
- 2 Lower demands on main memory bandwidth.

Two Main Goals of Cache

- Classification of Steady-State Cache Misses: *The Three C's of cache Misses*

- Cache Write Policies/Performance Evaluation:

- Write Though
- Write Back

- Cache Write Miss Policies: *Cache block allocation policy on a write miss.*

- Multi-Level Caches:

- Miss Rates For Multi-Level Caches
- 2-Level Cache Performance
- Write Policy For 2-Level Cache
- 3-Level Cache Performance

4th Edition: Chapter 5.1, Appendix C.1-C.3 (3rd Edition Chapter 5.1-5.4)

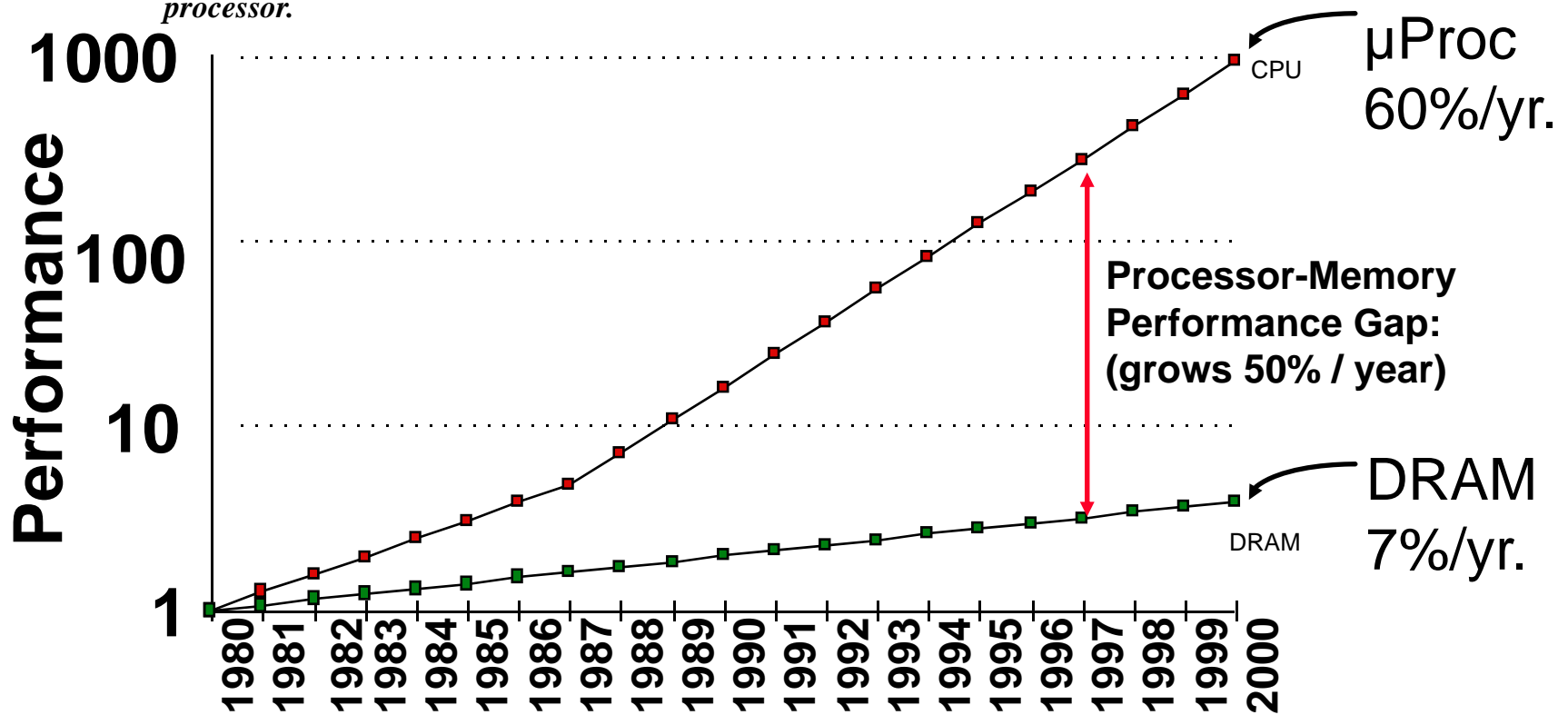
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Memory Hierarchy: Motivation

Processor-Memory (DRAM) Performance Gap

i.e. Gap between memory access time (latency) and CPU cycle time

Memory Access Latency: The time between a memory access request is issued by the processor and the time the requested information (instructions or data) is available to the processor.



Ideal Memory Access Time (latency) = 1 CPU Cycle
 Real Memory Access Time (latency) >> 1 CPU cycle

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Processor-DRAM Performance Gap Impact

- To illustrate the performance impact, assume a single-issue pipelined CPU with ideal CPI = 1 using non-ideal memory.
- Ignoring other factors, the minimum cost of a full memory access in terms of number of wasted CPU cycles: e.g cycles added to CPI

Year	CPU speed MHZ	CPU cycle ns	Memory Access ns	<u>Minimum CPU memory stall cycles</u> or instructions wasted
1986:	8	125	190	$190/125 - 1 = 0.5$
1989:	33	30	165	$165/30 - 1 = 4.5$
1992:	60	16.6	120	$120/16.6 - 1 = 6.2$
1996:	200	5	110	$110/5 - 1 = 21$
1998:	300	3.33	100	$100/3.33 - 1 = 29$
2000:	1000	1	90	$90/1 - 1 = 89$
2002:	2000	.5	80	$80/.5 - 1 = 159$
2004:	3000	.333	60	$60.333 - 1 = 179$

Or more
200+ Cycles

Ideal Memory Access Time (latency) = 1 CPU Cycle
Real Memory Access Time (latency) >> 1 CPU cycle

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Memory Access Latency Reduction & Hiding Techniques

Memory Latency Reduction Techniques:

Reduce it!

- Faster Dynamic RAM (DRAM) Cells: Depends on VLSI processing technology.
- Wider Memory Bus Width: Fewer memory bus accesses needed (e.g 128 vs. 64 bits)
- Multiple Memory Banks:
 - At DRAM chip level (SDR, DDR SDRAM), module or channel levels.
- Integration of Memory Controller with Processor: e.g AMD's processor architecture + Intel's i7
- New Emerging Faster RAM Technologies: e.g. Magnetoresistive Random Access Memory (MRAM)

Memory Latency Hiding Techniques:

Hide it!

→ – Memory Hierarchy: One or more levels of smaller and faster memory (SRAM-based cache) on- or off-chip that exploit program access locality to hide long main memory latency.

– Pre-Fetching: Request instructions and/or data from memory before actually needed to hide long memory access latency.

Get it from main memory into cache before you need it !

What about dynamic scheduling?

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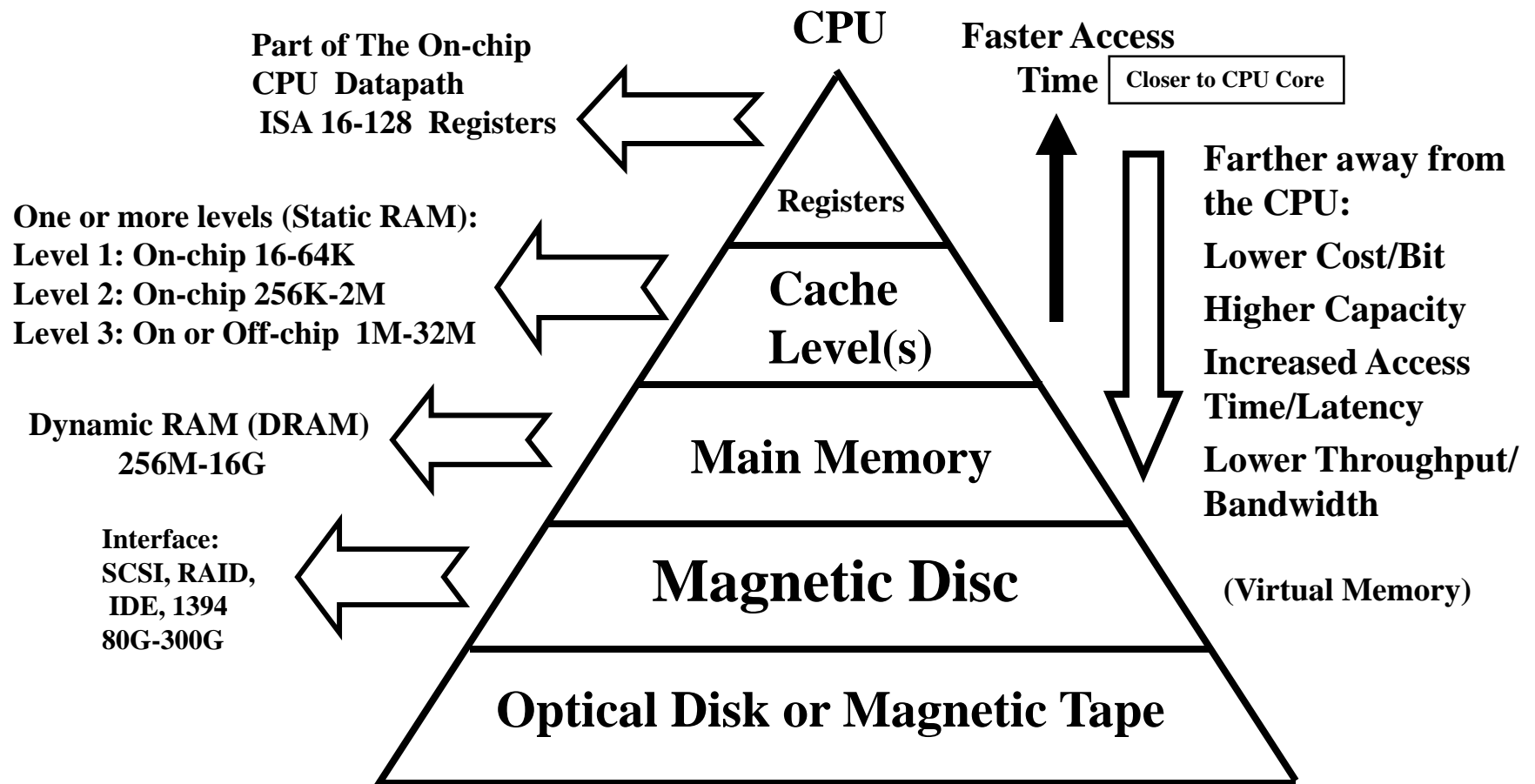
Memory Hierarchy: Motivation

- The gap between CPU performance and main memory has been widening with higher performance CPUs creating performance bottlenecks for memory access instructions.

For Ideal Memory: Memory Access Time or latency = 1 CPU cycle

- To hide long memory access latency, the memory hierarchy is organized into several levels of memory with the smaller, faster SRAM-based memory levels closer to the CPU: **registers**, then **primary Cache Level (L_1)**, then additional **secondary cache levels ($L_2, L_3...$)**, then **DRAM-based main memory**, then **mass storage** (virtual memory).
- Each level of the hierarchy is usually a subset of the level below: data found in a level is also found in the level below (farther from CPU) but at lower speed (longer access time).
- Each level maps addresses from a larger physical memory to a smaller level of physical memory closer to the CPU.
- This concept is greatly aided by the principal of locality both temporal and spatial which indicates that programs tend to reuse data and instructions that they have used recently or those stored in their vicinity leading to working set of a program.

Levels of The Memory Hierarchy



(Review from 350)

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Memory Hierarchy: Motivation

The Principle Of Locality

- Programs usually access a relatively small portion of their address space (instructions/data) at any instant of time (program working set).

Thus: Memory Access Locality → Program Working Set

- Two Types of access locality:

Often used data + instructions

1 – Temporal Locality: If an item (instruction or data) is referenced, it will tend to be referenced again soon.

- e.g. instructions in the body of inner loops

2 – Spatial locality: If an item is referenced, items whose addresses are close will tend to be referenced soon.

- e.g. sequential instruction execution, sequential access to elements of array



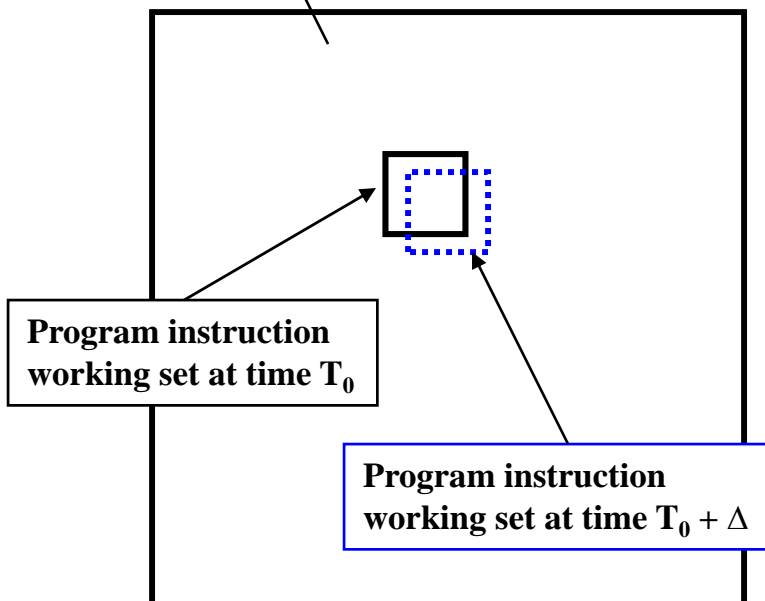
- The presence of locality in program behavior (memory access patterns), makes it possible to satisfy a large percentage of program memory access needs (both instructions and data) using faster memory levels (cache) with much less capacity than program address space.

Access Locality & Program Working Set

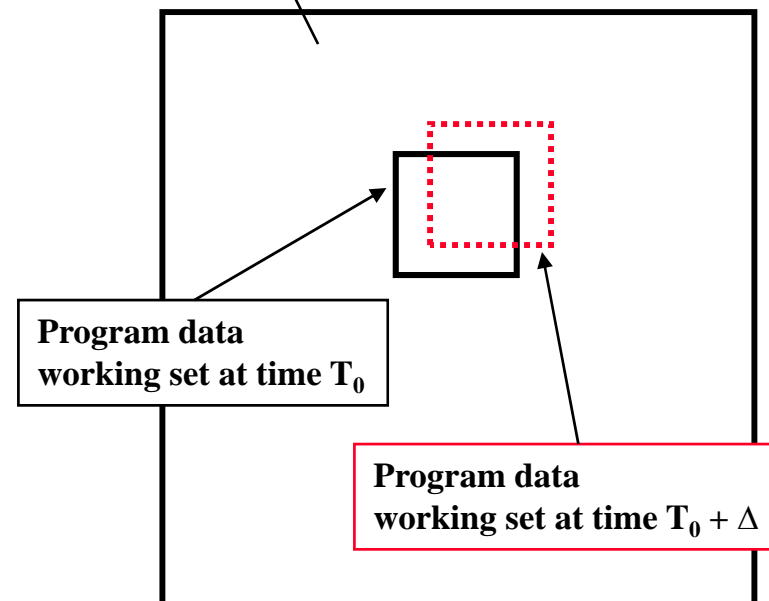
- Programs usually access a relatively small portion of their address space (instructions/data) at any instant of time (program working set).
- The presence of locality in program behavior and memory access patterns, makes it possible to satisfy a large percentage of program memory access needs using faster memory levels with much less capacity than program address space.
(i.e Cache)

Using Static RAM (SRAM)

Program Instruction Address Space



Program Data Address Space



Locality in program memory access → Program Working Set

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Memory Hierarchy Operation

- If an instruction or operand is required by the CPU, the levels of the memory hierarchy are searched for the item starting with the level closest to the CPU (Level 1 cache): L_1 Cache

Cache Hit

- If the item is found, it's delivered to the CPU resulting in a cache hit without searching lower levels.

Hit rate for level one cache = H_1

Cache Miss

- If the item is missing from an upper level, resulting in a cache miss, the level just below is searched.

Miss rate for level one cache = $1 - \text{Hit rate} = 1 - H_1$

- For systems with several levels of cache, the search continues with cache level 2, 3 etc.
- If all levels of cache report a miss then main memory is accessed for the item.
 - CPU \leftrightarrow cache \leftrightarrow memory: Managed by hardware.
- If the item is not found in main memory resulting in a page fault, then disk (virtual memory), is accessed for the item.
 - Memory \leftrightarrow disk: Managed by the operating system with hardware support

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Memory Hierarchy: Terminology

- **A Block:** The smallest unit of information transferred between two levels.
- **Hit:** Item is found in some block in the upper level (example: Block X)

e. g. H1

– **Hit Rate:** The fraction of memory access found in the upper level.

– **Hit Time:** Time to access the upper level which consists of

Ideally = 1 Cycle

Hit rate for level one cache = H_1

(S)RAM access time + Time to determine hit/miss

- **Miss:** Item needs to be retrieved from a block in the lower level (Block Y)

e. g. $1 - H_1$

– **Miss Rate** = $1 - (\text{Hit Rate})$

Miss rate for level one cache = $1 - \text{Hit rate} = 1 - H_1$

– **Miss Penalty:** Time to replace a block in the upper level +

M

Time to deliver the missed block to the processor

- **Hit Time** \ll **Miss Penalty M**

Ideally = 1 Cycle

Or Miss Time

CPU

To Processor
(Fetch/Load)

From Processor
(Store)

Level 1 (L_1) Cache

Upper Level Memory

Blk X



e.g cache

A block

Miss

Lower Level Memory

e.g main memory



Blk Y

M Stall cycles on a miss

Typical Cache Block (or line) Size: 16-64 bytes

Cache Hit if block is found in cache

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(Review from 350)

Basic Cache Concepts

- Cache is the first level of the memory hierarchy once the address leaves the CPU and is searched first for the requested data.
- If the data requested by the CPU is present in the cache, it is retrieved from cache and the data access is a cache hit otherwise a cache miss and data must be read from main memory.
- On a cache miss a block of data must be brought in from main memory to cache to possibly replace an existing cache block.
- The allowed block addresses where blocks can be mapped (placed) into cache from main memory is determined by cache placement strategy.
- Locating a block of data in cache is handled by cache block identification mechanism: Tag matching.
- On a cache miss choosing the cache block being removed (replaced) is handled by the block replacement strategy in place.
- When a write to cache is requested, a number of main memory update strategies exist as part of the cache write policy.

Basic Cache Design & Operation Issues

- Q1: Where can a block be placed cache?

Block placement

(Block placement strategy & Cache organization)

- Fully Associative, Set Associative, Direct Mapped.

Very complex

Most common

Simple but suffers from conflict misses

How many cache frames per set?

- Q2: How is a block found if it is in cache?

Locating a block

(Block identification)

Cache Hit/Miss?

- Tag/Block.

Tag Matching

- Q3: Which block should be replaced on a miss?

(Block replacement)

Block replacement

- Random, LRU, FIFO.

- Q4: What happens on a write?

(Cache write policy)

Not covered in 350 (Comp. Org.) will be covered here

- Write through, write back.

+ Cache block write allocation policy

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(Review from 350)

4th Edition: Appendix C.1 (3rd Edition Chapter 5.2)

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Cache Organization & Placement Strategies

How many cache frames per set? One = Direct Mapped (One-Way Set Associative) More than one = Set Associative All = Fully Associative

Placement strategies or mapping of a main memory data block onto cache block frames divide cache designs into three organizations:

1 **Direct mapped cache:** A block can be placed in only one location (cache block frame), given by the mapping function:

Least complex to implement
suffers from conflict misses

Mapping
Function

$$\text{index} = (\text{Block address}) \text{ MOD } (\text{Number of blocks in cache})$$

2 **Fully associative cache:** A block can be placed anywhere in cache. (no mapping function).

Most complex cache organization to implement

No Conflict
Misses

3 **Set associative cache:** A block can be placed in a restricted set of places, or cache block frames. A set is a group of block frames in the cache. A block is first mapped onto the set and then it can be placed anywhere within the set. The set in this case is chosen by:

Mapping
Function

$$\text{index} = (\text{Block address}) \text{ MOD } (\text{Number of sets in cache})$$

= Set #

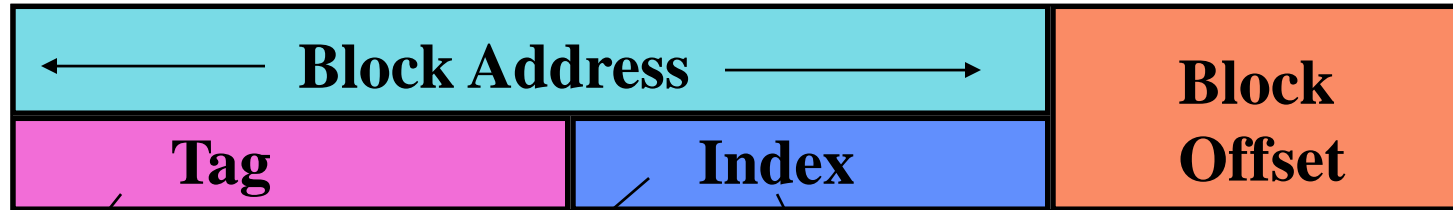
If there are n blocks in a set the cache placement is called n -way set-associative.

Most common cache organization

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Address Field Sizes/Mapping

← Physical Memory Address Generated by CPU →
 (size determined by amount of physical main memory cacheable)



Hit or Miss?
 Compare with tags in selected set in cache (tag matching)

Which Set ?

Mapping

Block offset size = $\log_2(\text{block size})$

Index size = $\log_2(\text{Total number of blocks/associativity})$

Tag size = address size - index size - offset size

Number of Sets in cache

Mapping function:

Cache set or block frame number = Index =
 = (Block Address) MOD (Number of Sets)

Mapping in cache is always done to a set

No index/mapping function for fully associative cache (it has just one set)

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Cache Organization: Direct Mapped Cache

Valid Bit

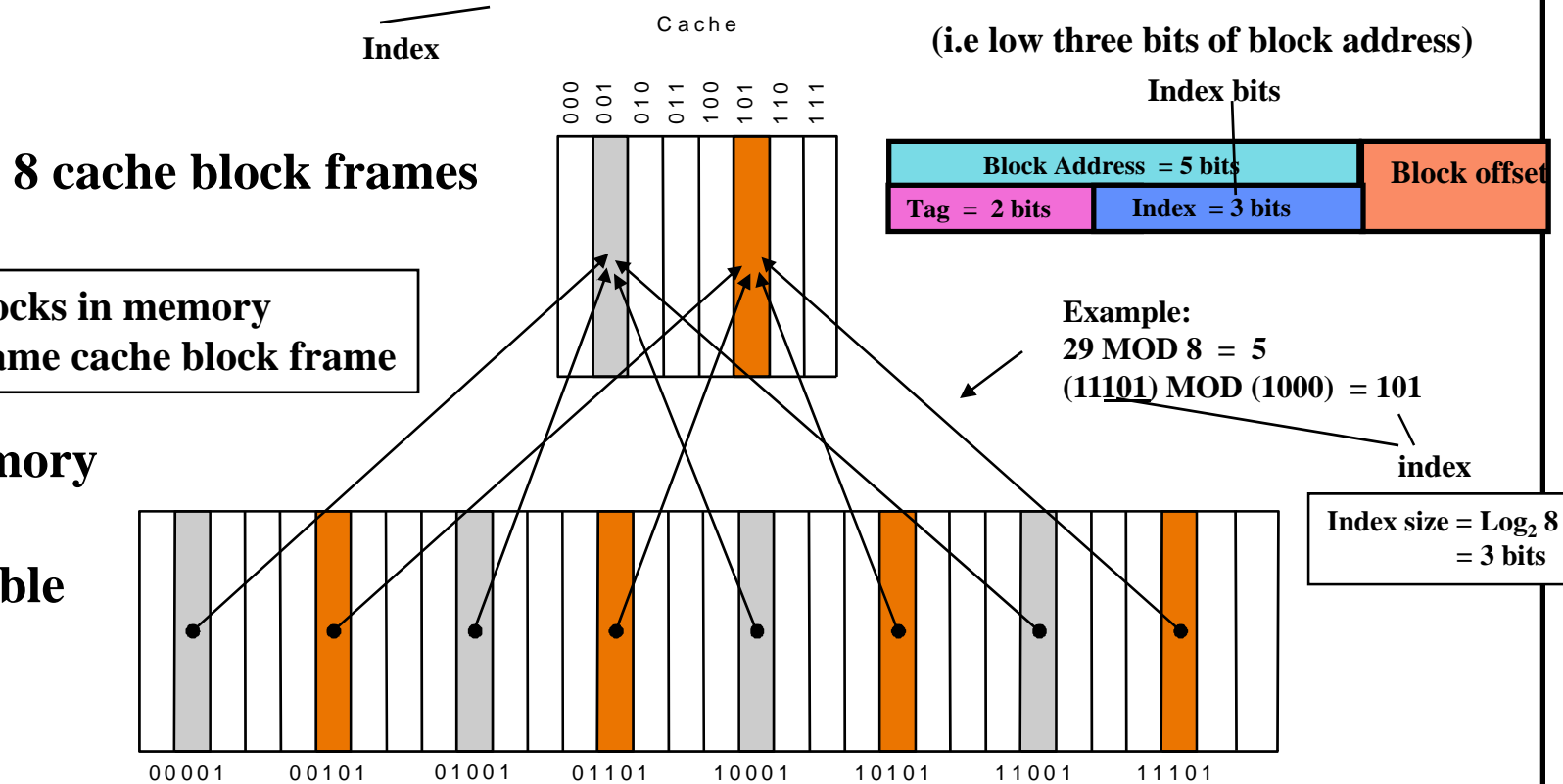


Cache Block Frame

A block can be placed in one location only, given by:

$(\text{Block address}) \text{ MOD } (\text{Number of blocks in cache})$

In this case, mapping function: $(\text{Block address}) \text{ MOD } (8) = \text{index}$



Here four blocks in memory map to the same cache block frame

Limitation of Direct Mapped Cache: Conflicts between memory blocks that map to the same cache block frame may result in conflict cache misses

4KB Direct Mapped Cache Example

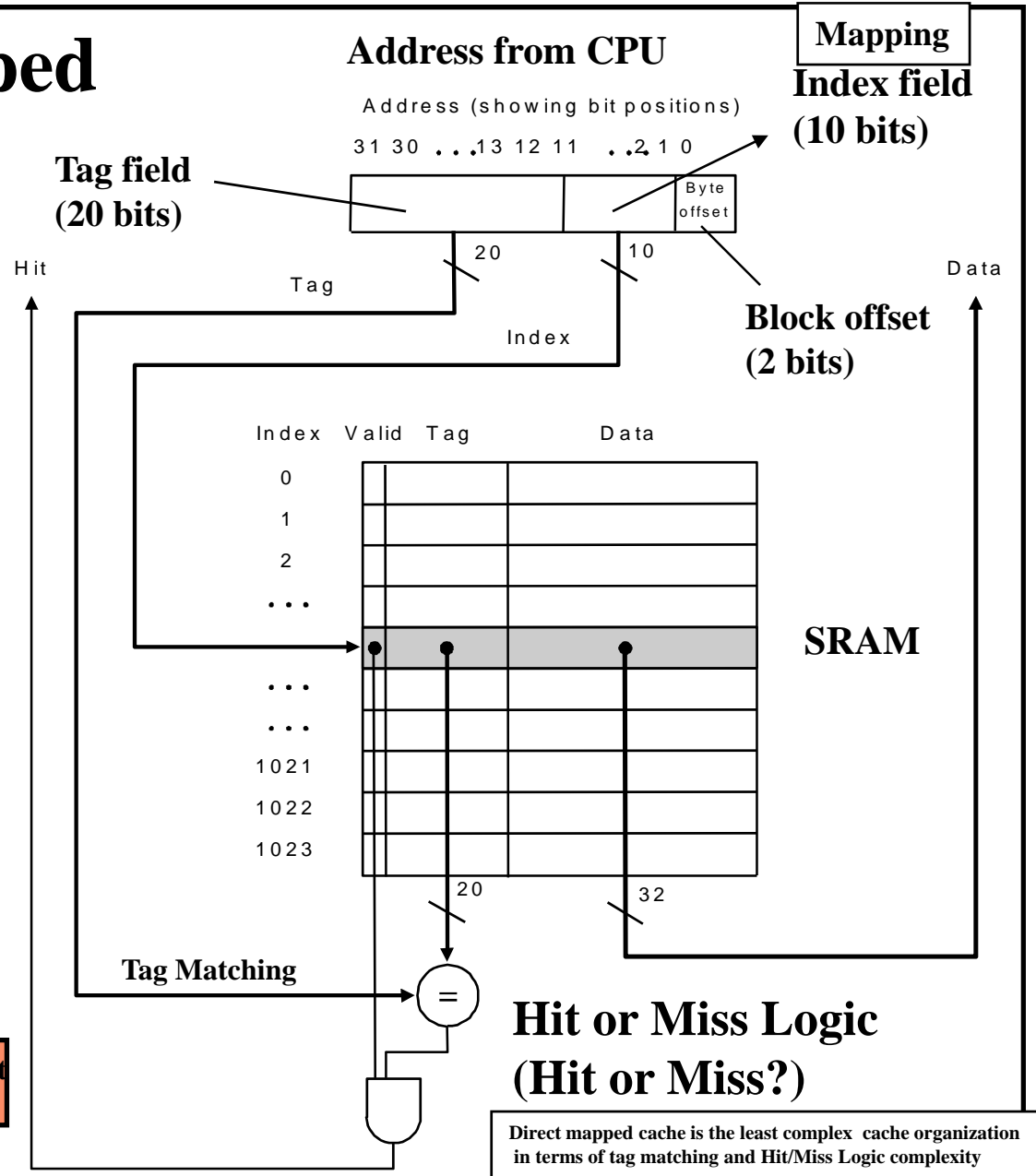
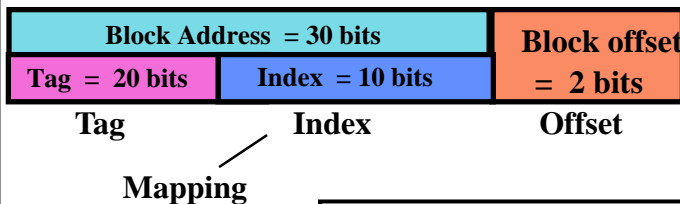
1K = 2^{10} = 1024 Blocks
 Each block = one word
 (4 bytes)

Can cache up to
 2^{32} bytes = 4 GB
 of memory

Mapping function:

Cache Block frame number =
 (Block address) MOD (1024)

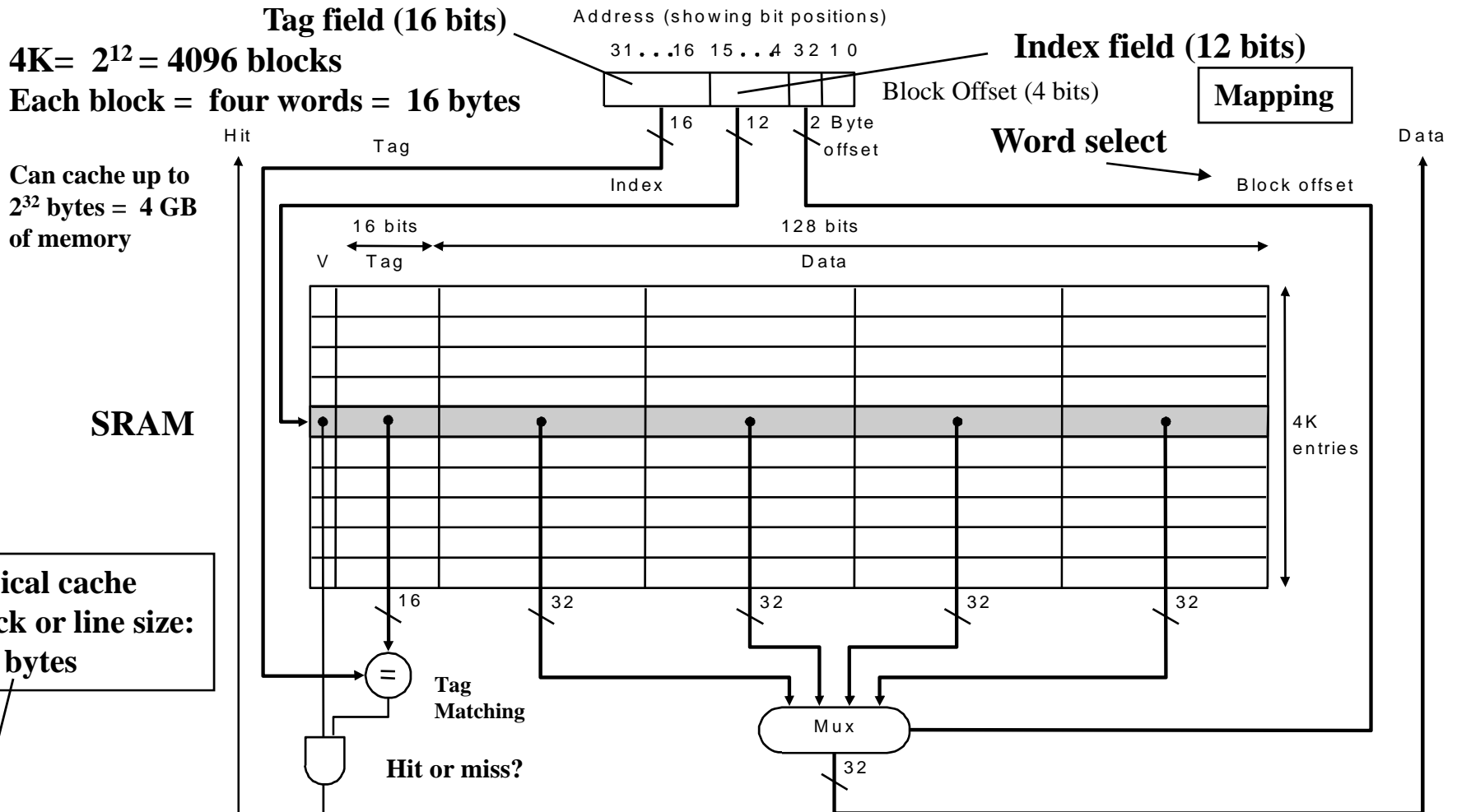
i.e. Index field or 10 low bits of
 block address



Hit Access Time = SRAM Delay + Hit/Miss Logic Delay

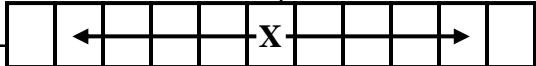
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64KB Direct Mapped Cache Example



Typical cache
Block or line size:
64 bytes

Larger cache blocks take better advantage of spatial locality and thus may result in a lower miss rate



Mapping Function: Cache Block frame number = (Block address) MOD (4096)
 i.e. index field or 12 low bit of block address

Block Address = 28 bits		Block offset = 4 bits
Tag = 16 bits	Index = 12 bits	

Hit Access Time = SRAM Delay + Hit/Miss Logic Delay

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(Review from 350)

Cache Organization:



Cache Block Frame

Set Associative Cache

Why set associative?

Set associative cache reduces cache misses by reducing conflicts between blocks that would have been mapped to the same cache block frame in the case of direct mapped cache

One-way set associative (direct mapped)

Block	Tag	Data
0		
1		
2		
3		
4		
5		
6		
7		

1-way set associative: (direct mapped) 1 block frame per set

Here 8 sets

Two-way set associative

Set	Tag	Data	Tag	Data
0				
1				
2				
3				

2-way set associative: 2 blocks frames per set

Here 4 sets

Four-way set associative

Set	Tag	Data	Tag	Data	Tag	Data	Tag	Data
0								
1								

4-way set associative: 4 blocks frames per set

Here 2 sets

8-way set associative: 8 blocks frames per set
In this case it becomes fully associative since total number of block frames = 8

Eight-way set associative (fully associative)

Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data

One set (no mapping)

A cache with a total of 8 cache block frames shown above

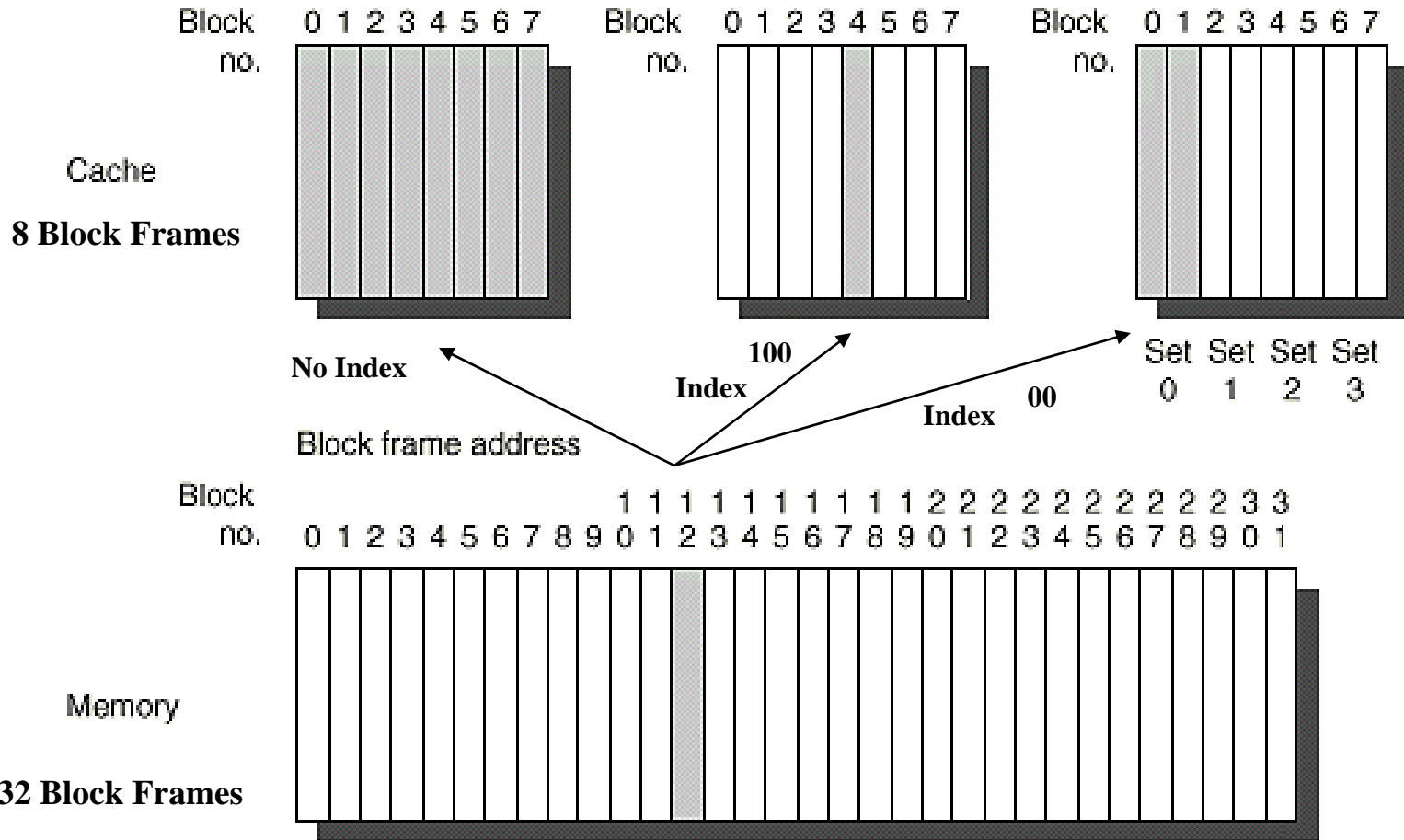
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Cache Organization/Mapping Example

Fully associative:
block 12 can go
anywhere
(No mapping function)

Direct mapped:
block 12 can go
only into block 4
(12 mod 8) = index = 100

2-way
Set associative:
block 12 can go
anywhere in set 0
(12 mod 4) = index = 00



This example cache has eight block frames and memory has 32 blocks.
12 = 1100

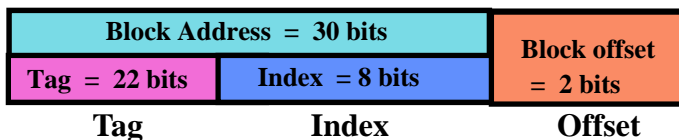
4K Four-Way Set Associative Cache: MIPS Implementation Example

Typically, primary or Level 1 (L1) cache is 2-8 way set associative

1024 block frames
Each block = one word
4-way set associative
 $1024 / 4 = 2^8 = 256$ sets

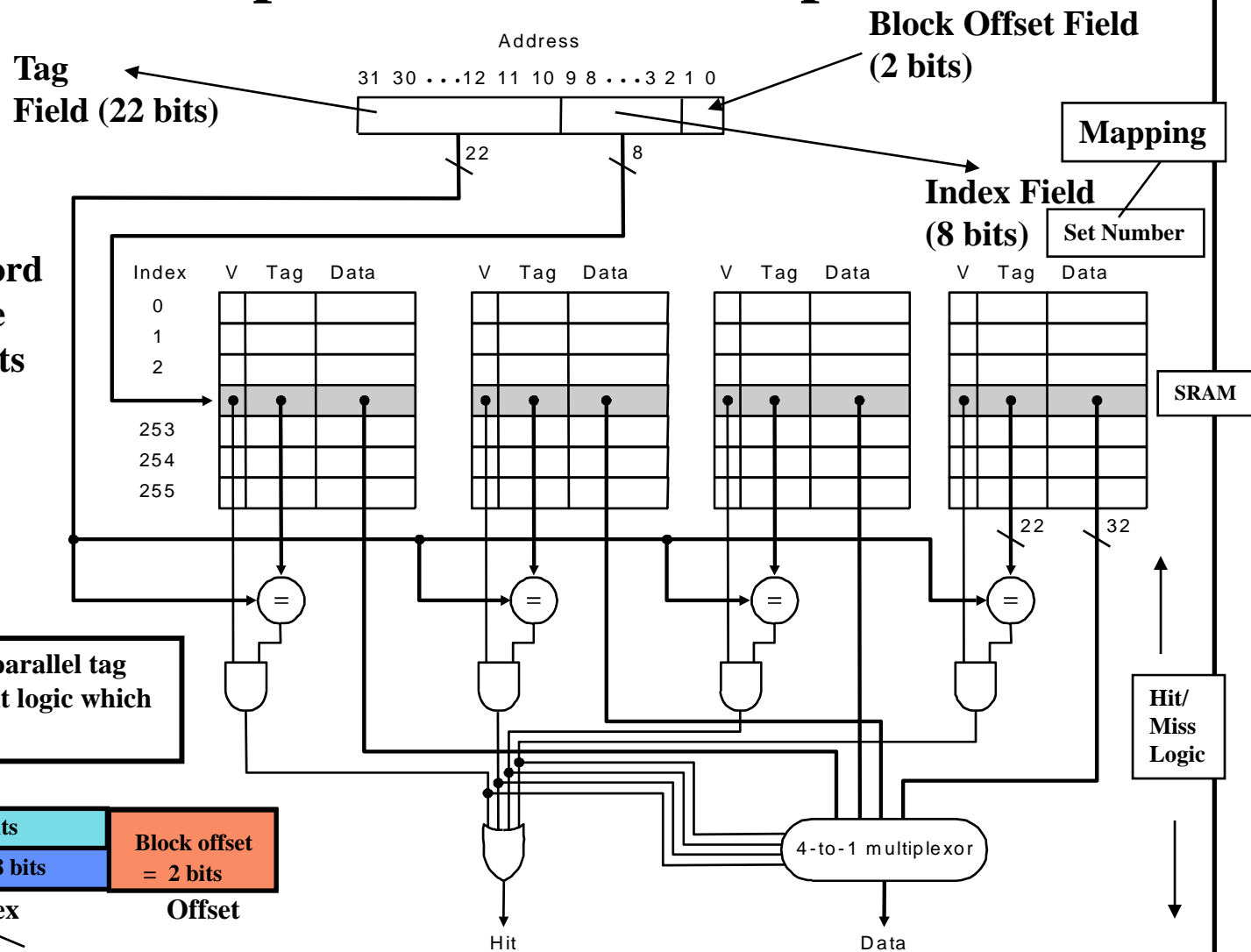
Can cache up to 2^{32} bytes = 4 GB of memory

Set associative cache requires parallel tag matching and more complex hit logic which may increase hit time



Mapping Function: Cache Set Number = index = (Block address) MOD (256)

Hit Access Time = SRAM Delay + Hit/Miss Logic Delay



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Locating A Data Block in Cache

- Each block frame in cache has an address tag.
- The tags of every cache block that might contain the required data are checked in parallel.
- A valid bit is added to the tag to indicate whether this entry contains a valid address.
- The address from the CPU to cache is divided into:
 - A block address, further divided into:
 - An index field to choose a block frame/set in cache.
(no index field when fully associative).
 - A tag field to search and match addresses in the selected set.
 - A block offset to select the data from the block.



Cache Replacement Policy

Which block to replace on a cache miss?

- When a cache miss occurs the cache controller may have to select a block of cache data to be removed from a cache block frame and replaced with the requested data, such a block is selected by one of three methods:

(No cache replacement policy in direct mapped cache)

No choice on which block to replace

1 – Random:

- Any block is randomly selected for replacement providing uniform allocation.
- Simple to build in hardware. Most widely used cache replacement strategy.

2 – Least-recently used (LRU): Optimal?

- Accesses to blocks are recorded and the block replaced is the one that was not used for the longest period of time.
- Full LRU is *expensive* to implement, as the number of blocks to be tracked increases, and is usually approximated by block usage (or activity) bits that are cleared at regular time intervals.

3 – First In, First Out (FIFO):

- Because LRU can be complicated to implement, this approximates LRU by determining the oldest block rather than LRU

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Miss Rates for Caches with Different Size, Associativity & Replacement Algorithm

Sample Data

Associativity:	2-way		4-way		8-way	
	LRU	Random	LRU	Random	LRU	Random
Size (Nominal)						
16 KB	5.18%	5.69%	4.67%	5.29%	4.39%	4.96%
64 KB	1.88%	2.01%	1.54%	1.66%	1.39%	1.53%
256 KB	1.15%	1.17%	1.13%	1.13%	1.12%	1.12%

Program steady state cache miss rates are given
Initially cache is empty and miss rates ~ 100%

All Compulsory Misses

FIFO replacement miss rates (not shown here) is better than random but worse than LRU

For SPEC92

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Unified vs. Separate Level 1 (L1) Cache

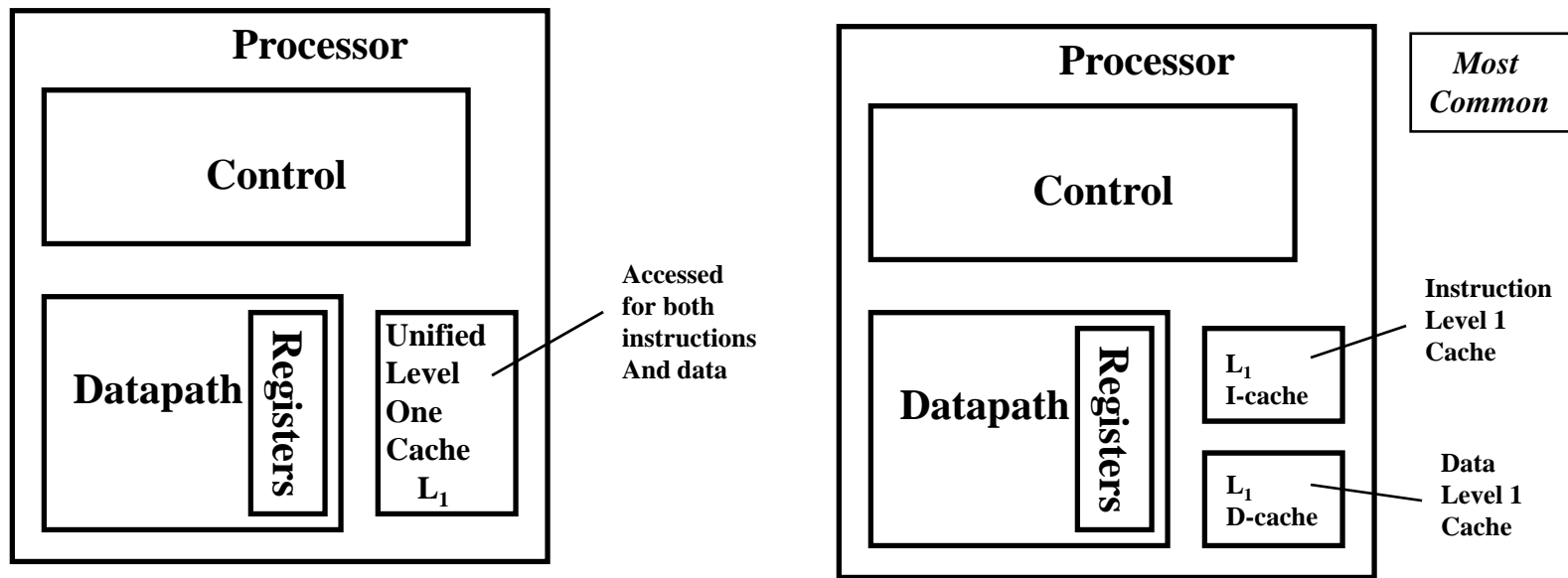
- Unified Level 1 Cache (Princeton Memory Architecture). AKA Shared Cache

A single level 1 (L_1) cache is used for both instructions and data.

Or Split

- Separate instruction/data Level 1 caches (Harvard Memory Architecture):

The level 1 (L_1) cache is split into two caches, one for instructions (instruction cache, L_1 I-cache) and the other for data (data cache, L_1 D-cache).



AKA shared

Unified Level 1 Cache
(Princeton Memory Architecture)

Separate (Split) Level 1 Caches
(Harvard Memory Architecture)

Why?

Split Level 1 Cache is more preferred in pipelined CPUs to avoid instruction fetch/Data access structural hazards

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Memory Hierarchy Performance:

Average Memory Access Time (AMAT), Memory Stall cycles

- The Average Memory Access Time (AMAT): The number of cycles required to complete an average memory access request by the CPU.
- Average memory stall cycles per memory access: The number of stall cycles added to CPU execution cycles for one memory access.

$$\text{Memory stall cycles per average memory access} = (\text{AMAT} - 1)$$

- For ideal memory: $\text{AMAT} = 1$ cycle, this results in zero memory stall cycles.

- Memory stall cycles per average instruction =

$$\begin{aligned} & \text{Number of memory accesses per instruction} \\ & \downarrow \\ \text{Instruction Fetch} & \rightarrow \text{Number of memory accesses per instruction} \times \text{Memory stall cycles per average memory access} \\ & = (1 + \text{fraction of loads/stores}) \times (\text{AMAT} - 1) \end{aligned}$$

$$\text{Base CPI} = \text{CPI}_{\text{execution}} = \text{CPI with ideal memory}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + \text{Mem Stall cycles per instruction}$$

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cycles = CPU cycles

Cache Performance:

(Ignoring Write Policy)

Single Level L1 Princeton (Unified) Memory Architecture

CPUtime = Instruction count x CPI x Clock cycle time

(Review from 350)

$CPI_{\text{execution}} = \text{CPI with ideal memory}$

$$CPI = CPI_{\text{execution}} + \text{Mem Stall cycles per instruction}$$

Mem Stall cycles per instruction =

Memory accesses per instruction x Memory stall cycles per access

Assuming no stall cycles on a cache hit (cache access time = 1 cycle, stall = 0)

Cache Hit Rate = H1 Miss Rate = 1 - H1

i.e No hit penalty

Stalls on
a Miss

Memory stall cycles per memory access = Miss rate x Miss penalty = (1 - H1) x M

AMAT = 1 + Miss rate x Miss penalty = 1 + (1 - H1) x M

+ H1 x 0

Memory accesses per instruction = (1 + fraction of loads/stores)

Miss Penalty = M = the number of stall cycles resulting from missing in cache
= Main memory access time - 1

Thus for a unified L1 cache with no stalls on a cache hit:

$$CPI = CPI_{\text{execution}} + (1 + \text{fraction of loads/stores}) \times (1 - H1) \times M$$
$$AMAT = 1 + (1 - H1) \times M$$

$$CPI = CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access}$$
$$= CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (AMAT - 1)$$

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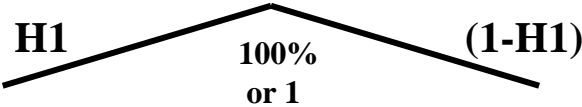
Memory Access Tree: For Unified Level 1 Cache

(Ignoring Write Policy)

Probability to be here

CPU Memory Access

Unified
L₁



L1 Hit:

% = Hit Rate = H1
 Hit Access Time = 1
 Stall cycles per access = 0
 Stall = H1 x 0 = 0
 (No Stall)

L1 Miss:

% = (1- Hit rate) = (1-H1)
 Access time = M + 1
 Stall cycles per access = M
 Stall = M x (1-H1)

Assuming:
Ideal access on a hit

$$AMAT = \overset{\text{Hit Rate}}{H1} \times \overset{\text{Hit Time}}{1} + \overset{\text{Miss Rate}}{(1-H1)} \times \overset{\text{Miss Time}}{(M+1)} = 1 + M \times (1-H1)$$

$$\text{Stall Cycles Per Access} = AMAT - 1 = M \times (1-H1) \quad [+ H1 \times 0]$$

$$CPI = CPI_{\text{execution}} + (1 + \text{fraction of loads/stores}) \times M \times (1-H1)$$

M = Miss Penalty = stall cycles per access resulting from missing in cache
 M + 1 = Miss Time = Main memory access time
 H1 = Level 1 Hit Rate 1- H1 = Level 1 Miss Rate

(Review from 350)

$$AMAT = 1 + \text{Stalls per average memory access}$$

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Cache Performance Example

- Suppose a CPU executes at Clock Rate = 200 MHz (5 ns per cycle) with a single level of cache.
- $CPI_{\text{execution}} = 1.1$ (i.e base CPI with ideal memory)
- Instruction mix: 50% arith/logic, 30% load/store, 20% control
- Assume a cache miss rate of 1.5% and a miss penalty of $M= 50$ cycles.

$$CPI = CPI_{\text{execution}} + \text{mem stalls per instruction} \quad \text{i.e. Stall Cycles on a cache miss}$$

Mem Stalls per instruction =

$$\text{Mem accesses per instruction} \times \text{Miss rate} \times \text{Miss penalty}$$

$$\text{Mem accesses per instruction} = 1 + .3 = 1.3$$

Instruction fetch

Load/store

$$\text{Mem Stalls per memory access} = (1 - H1) \times M = .015 \times 50 = .75 \text{ cycles}$$

$$AMAT = 1 + .75 = 1.75 \text{ cycles}$$

$$+ H1 \times 0$$

$$\text{Mem Stalls per instruction} = 1.3 \times .015 \times 50 = 0.975$$

$$CPI = 1.1 + .975 = 2.075$$

The ideal memory CPU with no misses is $2.075/1.1 = 1.88$ times faster

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M = Miss Penalty = stall cycles per access resulting from missing in cache

Cache Performance Example

- Suppose for the previous example we double the clock rate to 400 MHz, how much faster is this machine, assuming similar miss rate, instruction mix?
- Since memory speed is not changed, the miss penalty takes more CPU cycles:

$$\text{Miss penalty} = M = 50 \times 2 = 100 \text{ cycles.}$$

$$\text{CPI} = 1.1 + 1.3 \times .015 \times 100 = 1.1 + 1.95 = 3.05$$

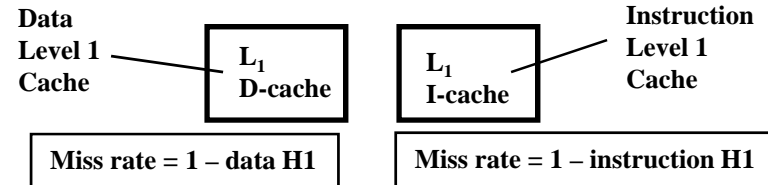
$$\begin{aligned} \text{Speedup} &= (\text{CPI}_{\text{old}} \times C_{\text{old}}) / (\text{CPI}_{\text{new}} \times C_{\text{new}}) \\ &= 2.075 \times 2 / 3.05 = 1.36 \end{aligned}$$

The new machine is only 1.36 times faster rather than 2 times faster due to the increased effect of cache misses.

→ *CPUs with higher clock rate, have more cycles per cache miss and more memory impact on CPI.*

(Ignoring Write Policy)

Usually:
Data Miss Rate >> Instruction Miss Rate



Cache Performance:

Single Level L1 Harvard (Split) Memory Architecture

For a CPU with separate or split level one (L1) caches for instructions and data (Harvard memory architecture) and no stalls for cache hits:

$$\text{CPUtime} = \text{Instruction count} \times \text{CPI} \times \text{Clock cycle time}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + \text{Mem Stall cycles per instruction}$$

As assumption
In reality, even a hit may result in few stalls (2-3)

Mem Stall cycles per instruction =

This is one method to find stalls per instruction another method is shown in next slide →

$$\text{Mem Stall cycles per instruction} = \text{Instruction Fetch Miss rate} \times M + \text{Data Memory Accesses Per Instruction} \times \text{Data Miss Rate} \times M$$

1- Instruction H1

Fraction of Loads and Stores

1- Data H1

M = Miss Penalty = stall cycles per access to main memory resulting from missing in cache

$$\text{CPI}_{\text{execution}} = \text{base CPI with ideal memory}$$

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Memory Access Tree

For Separate Level 1 Caches

(Ignoring Write Policy)

Split
L₁

CPU Memory Access

1 or 100%

% Instructions

% data

Instruction

Data

%instructions x
Instruction H1)

%instructions
x (1 - Instruction H1)

% data x Data H1

% data x (1 - Data H1)

Instruction L1 Hit:
Hit Access Time = 1
Stalls = 0

Instruction L1 Miss:
Access Time = M + 1
Stalls Per access = M

Data L1 Hit:
Hit Access Time: = 1
Stalls = 0

Data L1 Miss:
Access Time = M + 1
Stalls per access: M
Stalls = % data x (1 - Data H1) x M

Assuming:
Ideal access on a hit, no stalls

Assuming:
Ideal access on a hit, no stalls

Stall Cycles Per Access = % Instructions x (1 - Instruction H1) x M + % data x (1 - Data H1) x M

AMAT = 1 + Stall Cycles per access

Stall cycles per instruction = (1 + fraction of loads/stores) x Stall Cycles per access

CPI = CPI_{execution} + Stall cycles per instruction

= CPI_{execution} + (1 + fraction of loads/stores) x Stall Cycles per access

M = Miss Penalty = stall cycles per access resulting from missing in cache

M + 1 = Miss Time = Main memory access time

Data H1 = Level 1 Data Hit Rate

1 - Data H1 = Level 1 Data Miss Rate

Instruction H1 = Level 1 Instruction Hit Rate

1 - Instruction H1 = Level 1 Instruction Miss Rate

% Instructions = Percentage or fraction of instruction fetches out of all memory accesses

% Data = Percentage or fraction of data accesses out of all memory accesses

(Review from 350)

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Split L1 Cache Performance Example

- Suppose a CPU uses separate level one (L1) caches for instructions and data (Harvard memory architecture) with different miss rates for instruction and data access:
 - A cache hit incurs no stall cycles while a cache miss incurs 200 stall cycles for both memory reads and writes.
 - $CPI_{\text{execution}} = 1.1$ (i.e base CPI with ideal memory)
 - Instruction mix: 50% arith/logic, 30% load/store, 20% control
 - Assume a cache miss rate of 0.5% for instruction fetch and a cache data miss rate of 6%.
 - A cache hit incurs no stall cycles while a cache miss incurs 200 stall cycles for both memory reads and writes.

- Find the resulting stalls per access, AMAT and CPI using this cache?

M

(Ignoring Write Policy)

$$CPI = CPI_{\text{execution}} + \text{mem stalls per instruction}$$

$$\text{Memory Stall cycles per instruction} = \text{Instruction Fetch Miss rate} \times \text{Miss Penalty} + \text{Data Memory Accesses Per Instruction} \times \text{Data Miss Rate} \times \text{Miss Penalty}$$

$$\text{Memory Stall cycles per instruction} = 0.5/100 \times 200 + 0.3 \times 6/100 \times 200 = 1 + 3.6 = 4.6 \text{ cycles}$$

$$\text{Stall cycles per average memory access} = 4.6/1.3 = 3.54 \text{ cycles}$$

$$AMAT = 1 + 3.54 = 4.54 \text{ cycles}$$

$$CPI = CPI_{\text{execution}} + \text{mem stalls per instruction} = 1.1 + 4.6 = 5.7 \text{ cycles}$$

- What is the miss rate of a single level unified cache that has the same performance?

$$4.6 = 1.3 \times \text{Miss rate} \times 200 \quad \text{which gives a miss rate of 1.8 \% for an equivalent unified cache}$$

- How much faster is the CPU with ideal memory?

The CPU with ideal cache (no misses) is $5.7/1.1 = 5.18$ times faster

With no cache at all the CPI would have been = $1.1 + 1.3 \times 200 = 261.1$ cycles !!

Memory Access Tree For Separate Level 1 Caches Example

(Ignoring Write Policy)

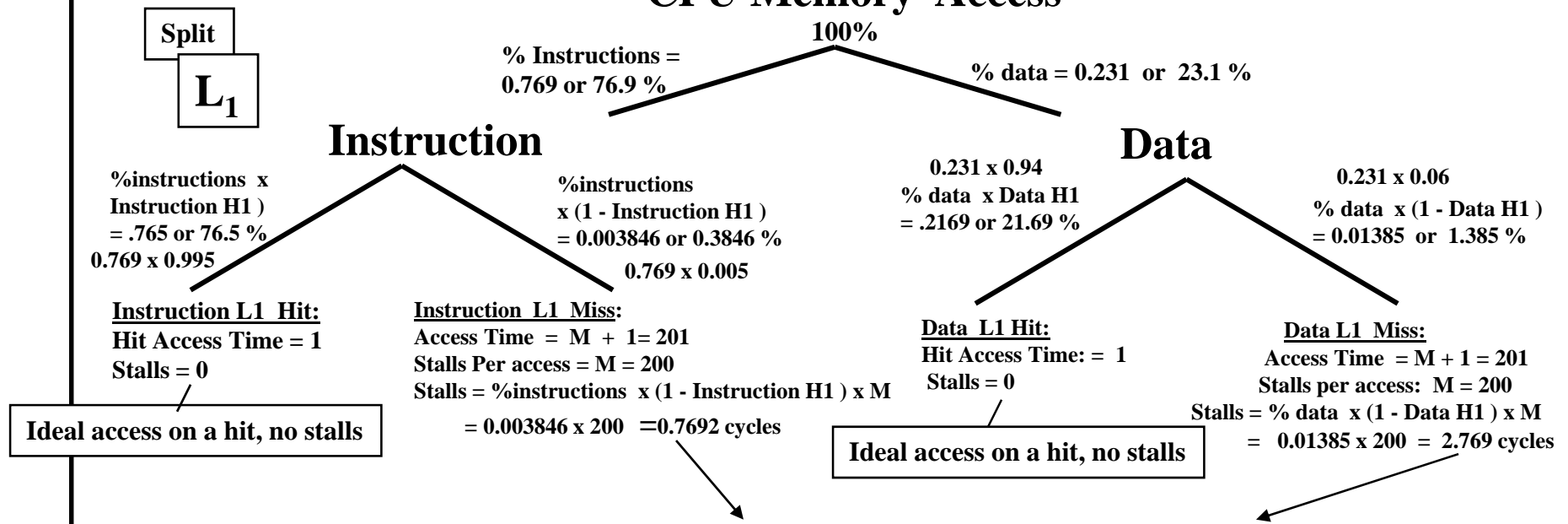
30% of all instructions executed are loads/stores, thus:

Fraction of instruction fetches out of all memory accesses = $1 / (1+0.3) = 1/1.3 = 0.769$ or 76.9 %

Fraction of data accesses out of all memory accesses = $0.3 / (1+0.3) = 0.3/1.3 = 0.231$ or 23.1 %

For Last Example

CPU Memory Access



$$\text{Stall Cycles Per Access} = \% \text{ Instructions} \times (1 - \text{Instruction H1}) \times M + \% \text{ data} \times (1 - \text{Data H1}) \times M$$

$$= 0.7692 + 2.769 = 3.54 \text{ cycles}$$

$$\text{AMAT} = 1 + \text{Stall Cycles per access} = 1 + 3.5 = 4.54 \text{ cycles}$$

$$\text{Stall cycles per instruction} = (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access} = 1.3 \times 3.54 = 4.6 \text{ cycles}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + \text{Stall cycles per instruction} = 1.1 + 4.6 = 5.7$$

Given as 1.1

M = Miss Penalty = stall cycles per access resulting from missing in cache = 200 cycles
 M + 1 = Miss Time = Main memory access time = 200+1 = 201 cycles L1 access Time = 1 cycle
 Data H1 = 0.94 or 94% 1- Data H1 = 0.06 or 6%
 Instruction H1 = 0.995 or 99.5% 1- Instruction H1 = 0.005 or 0.5 %
 % Instructions = Percentage or fraction of instruction fetches out of all memory accesses = 76.9 %
 % Data = Percentage or fraction of data accesses out of all memory accesses = 23.1 %

(Review from 350)

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Typical Cache Performance Data Using SPEC92

Usually Data Miss Rate >> Instruction Miss Rate

Size	Instruction cache	Data cache	Unified cache
1 KB	3.06%	24.61%	13.34%
2 KB	2.26%	20.57%	9.78%
4 KB	1.78%	15.94%	7.24%
8 KB	1.10%	10.19%	4.57%
16 KB	0.64%	6.47%	2.87%
32 KB	0.39%	4.82%	1.99%
64 KB	0.15%	3.77%	1.35%
128 KB	0.02%	2.88%	0.95%

Miss rates for instruction, data, and unified caches of different sizes.

Program steady state cache miss rates are given
Initially cache is empty and miss rates ~ 100%

All Compulsory Misses

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Types of Cache Misses: *The Three C's*

(of Cache Misses)

1 Compulsory: On the first access to a block; the block must be brought into the cache; also called cold start misses, or first reference misses.

- Initially upon program startup: Miss rate ~ 100% All compulsory misses

Can be reduced by increasing cache block size and pre-fetching

2 Capacity: Occur because blocks are being discarded from cache because cache cannot contain all blocks needed for program execution (program working set is much larger than cache capacity).

i.e. cache filled

Can be reduced by increasing total cache size

3 Conflict: In the case of set associative or direct mapped block placement strategies, conflict misses occur when several blocks are mapped to the same set or block frame; also called collision misses or interference misses.

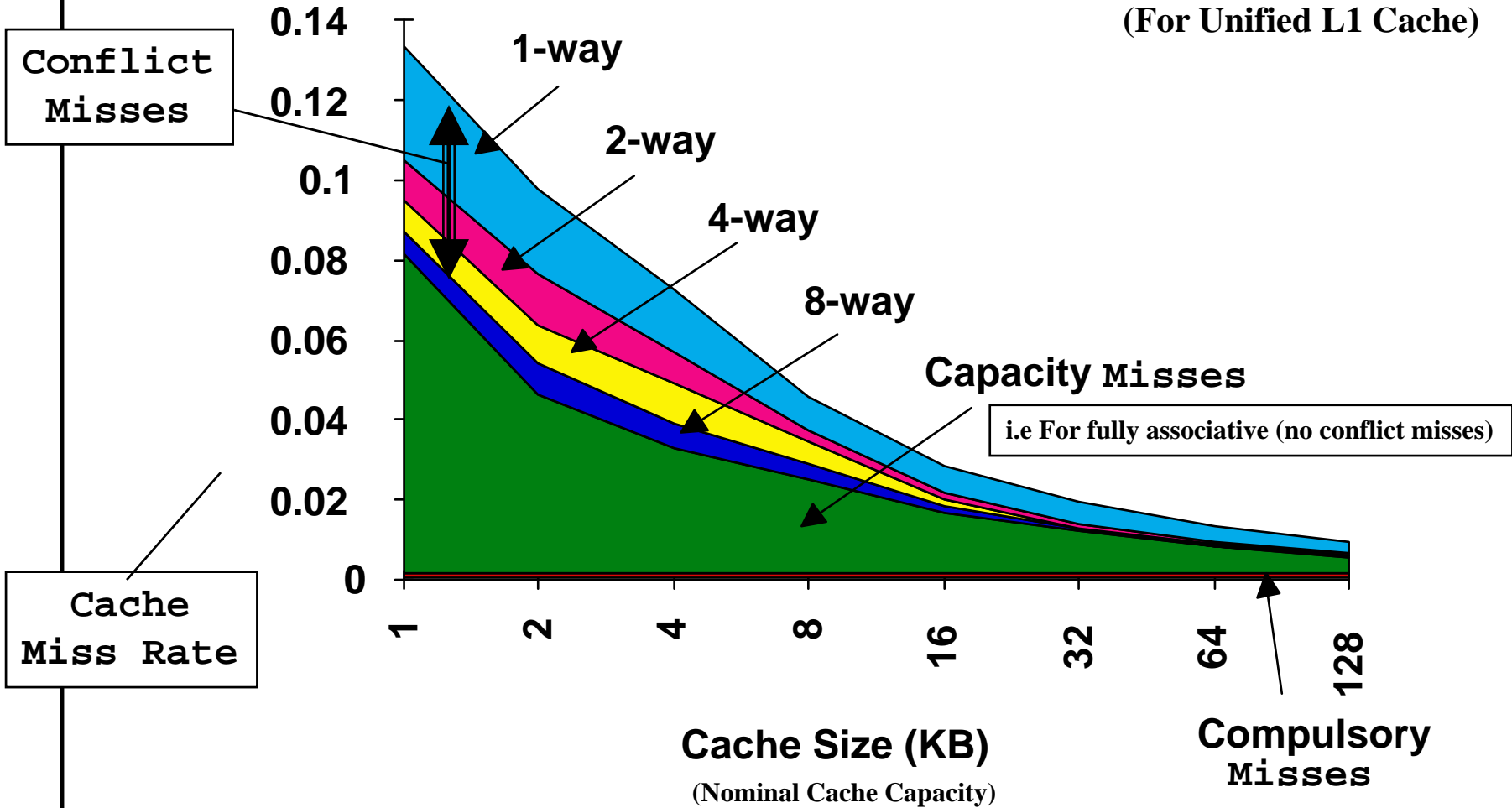
Can be reduced by increasing cache associativity
+ Better Replacement Policy ?

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The 3 Cs of Cache:

Absolute Steady State Miss Rates (SPEC92)

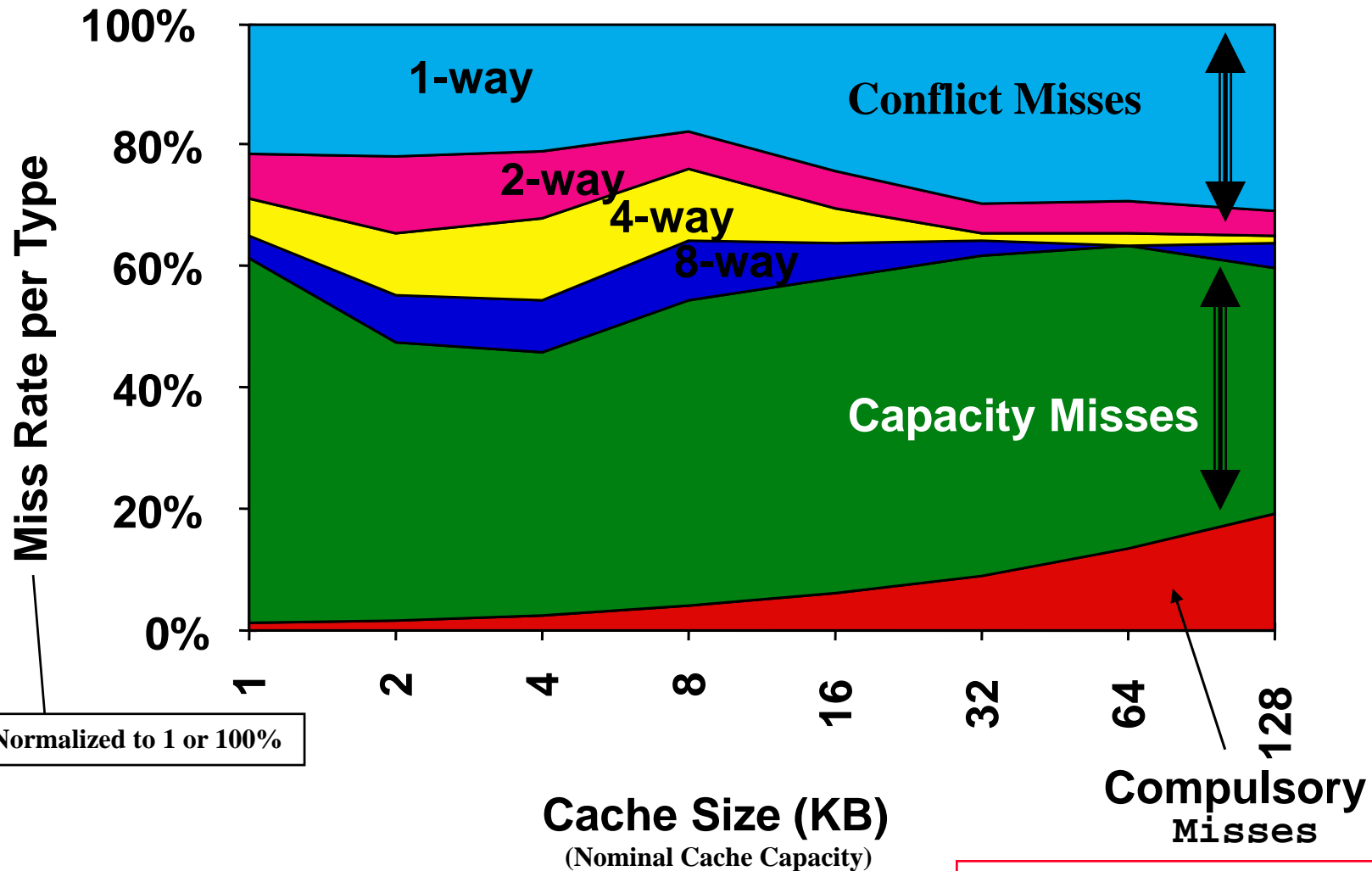
(For Unified L1 Cache)



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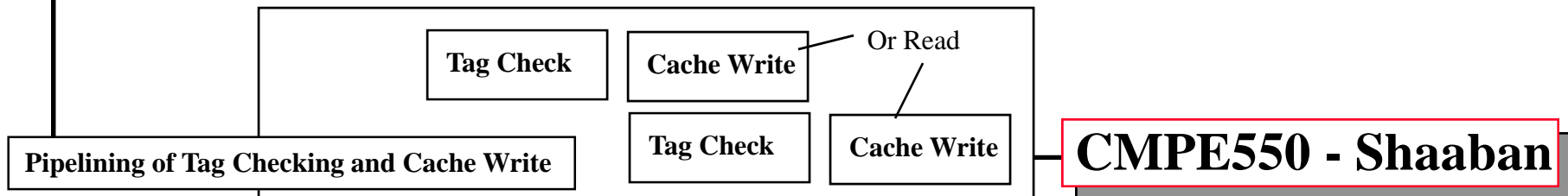
The 3 Cs of Cache:

Relative Steady State Miss Rates (SPEC92)



Cache Read/Write Operations

- Statistical data suggest that reads (*including instruction fetches*) dominate processor cache accesses (writes account for ~ 25% of data cache traffic).
i.e stores
- In cache reads, a block is read at the same time while the tag is being compared with the block address. If the read is a hit the data is passed to the CPU, if a miss it ignores it.
- In cache writes, modifying the block cannot begin until the tag is checked to see if the address is a hit.
i.e write hit (we have old block to modify in cache)
- Thus for cache writes, tag checking cannot take place in parallel, and only the specific data (between 1 and 8 bytes) requested by the CPU can be modified.
 - Solution: Pipeline tag checking and cache write.
- Cache can be classified according to the write and memory update strategy in place as: write through, or write back cache.



Cache Write (For Stores) Strategies

1 **Write Through**: Data is written to both the cache block and to a block of main memory. (i.e written though to memory)

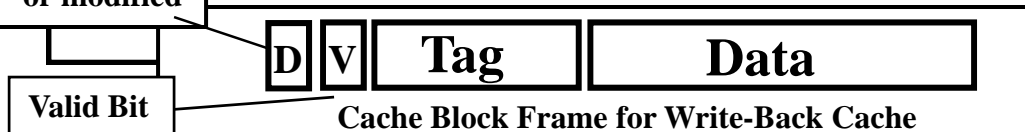
- The lower level always has the most updated data; an important feature for I/O and multiprocessing.
- Easier to implement than write back.
- A write buffer is often used to reduce CPU write stall while data is written to memory.

The updated cache block is marked as modified or dirty

2 **Write Back**: Data is written or updated only to the cache block. The modified or dirty cache block is written to main memory when it's being replaced from cache. back

- Writes occur at the speed of cache
- A status bit called a dirty or modified bit, is used to indicate whether the block was modified while in cache; if not the block is not written back to main memory when replaced. i.e discarded
- Advantage: Uses less memory bandwidth than write through.

D = Dirty
Or
Modified
Status Bit
0 = clean
1 = dirty
or modified



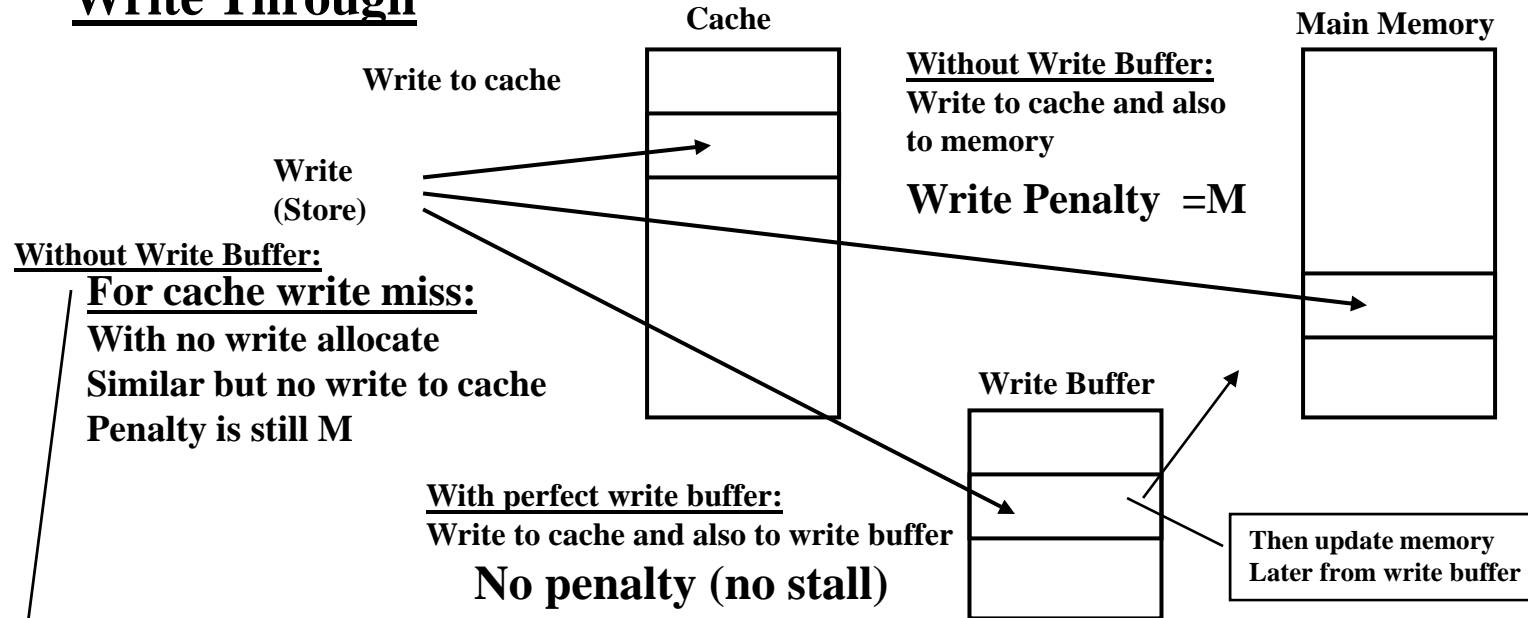
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Cache Write Strategies:

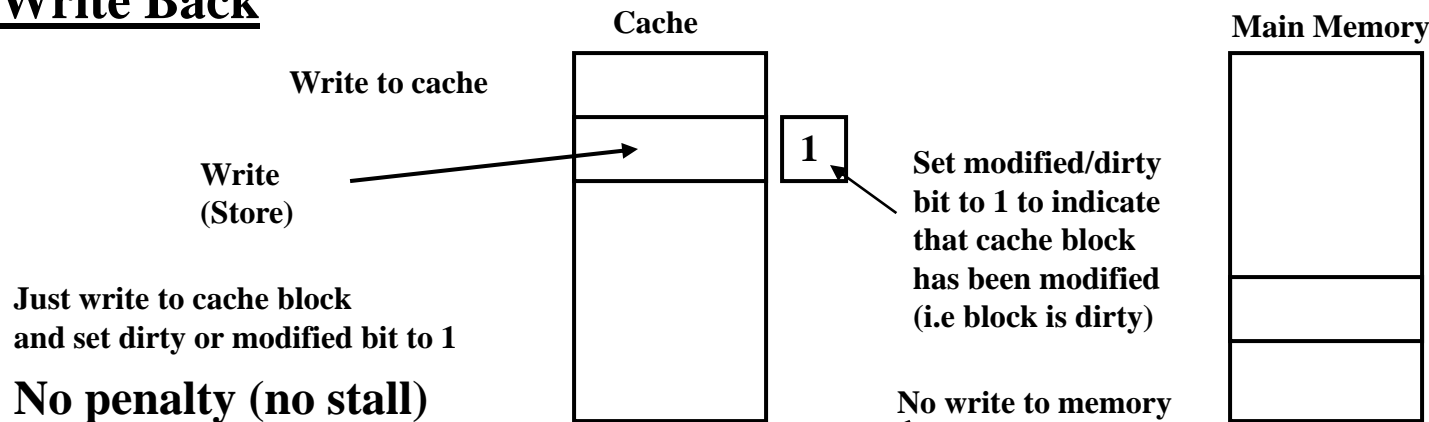
Cache Write Hit = block to be modified is found in cache

Write Hit Operation (block to be written to is in cache)

Write Through



Write Back



Write back to memory when replaced in cache

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Cache Write Miss = block to be modified is not in cache

Cache Write Miss Policy

- Since data is usually not needed immediately on a write miss two options exist on a cache write miss:

1

Write Allocate: *(Bring old block to cache then update it)*

The missed cache block is loaded into cache on a write miss followed by write hit actions.

i.e. A cache block frame is allocated for the block to be modified (written-to)

2

No-Write Allocate:

i.e. A cache block frame is not allocated for the block to be modified (written-to)

The block is modified in the lower level (lower cache level, or main memory) and not loaded (written or updated) into cache.

While any of the above two write miss policies can be used with either write back or write through:

- Write back caches always use write allocate to capture subsequent writes to the block in cache.
- Write through caches usually use no-write allocate since subsequent writes still have to go to memory.

Cache Write Miss = Block to be modified is not in cache
Allocate = Allocate or assign a cache block frame for written data

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Write Back Cache With Write Allocate: Cache Miss Operation

(read or write miss)

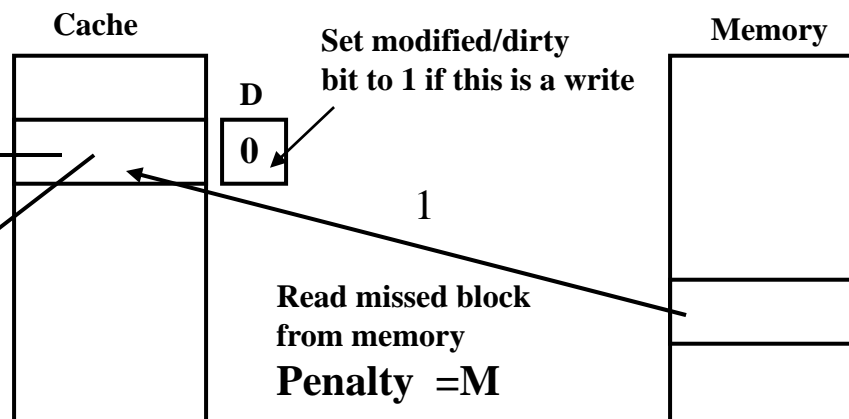
Block to be replaced is clean

Miss Penalty = M

i.e. D was = 0

CPU reads or writes to block in cache

Replaced (old) block is discarded since it's clean



Block to be replaced is dirty (modified)

- 1 Write back modified block being replaced to memory
Penalty = M

i.e. D was = 1

- 2 Read missed block from memory
Penalty = M

Thus:

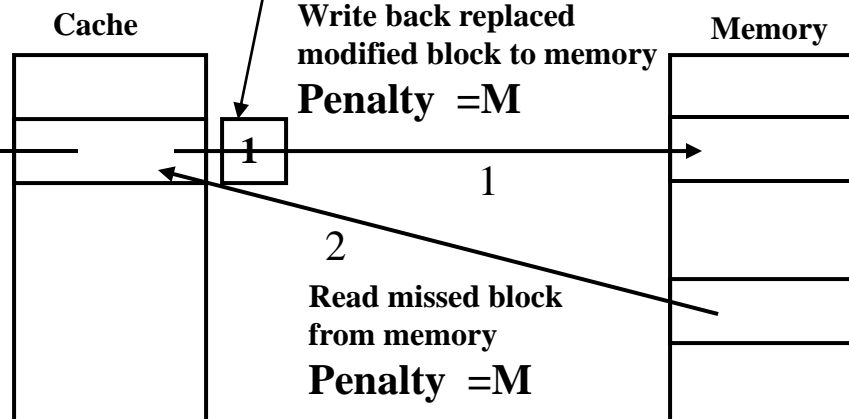
Total Miss Penalty = M + M = 2M

CPU reads or writes to block in cache

Set modified/dirty bit to 1 if this is a write

Write back replaced modified block to memory
Penalty = M

Read missed block from memory
Penalty = M



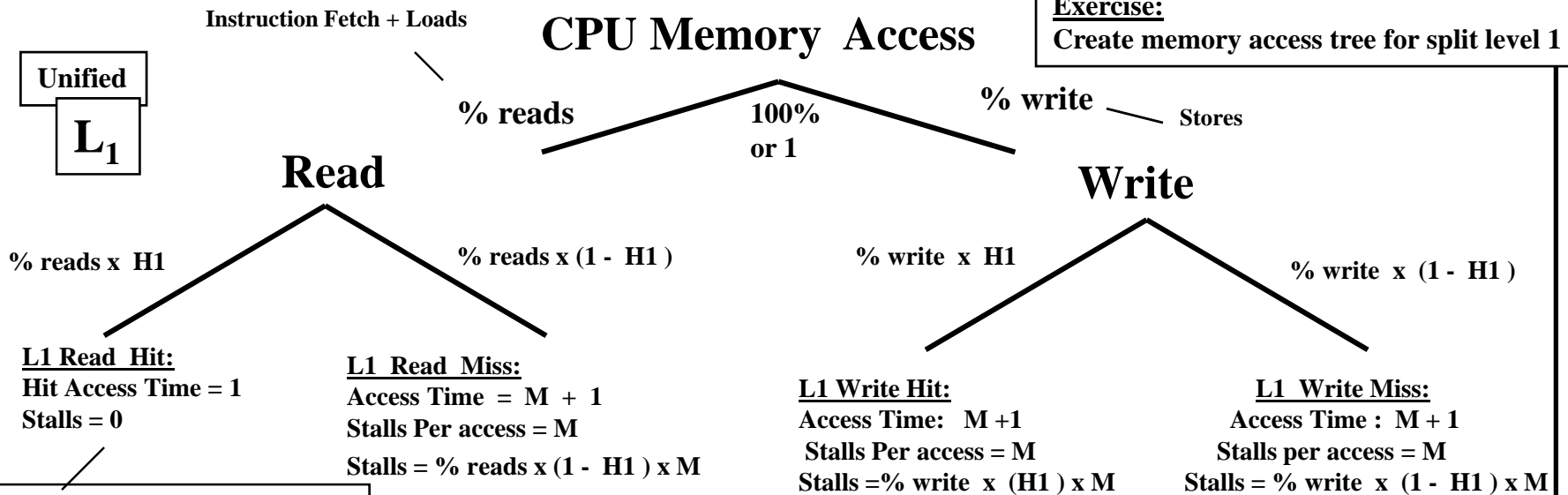
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M = Miss Penalty = stall cycles per access resulting from missing in cache

Memory Access Tree, Unified L₁

Write Through, No Write Allocate, No Write Buffer

Exercise:
Create memory access tree for split level 1



Assuming:
Ideal access on a read hit, no stalls

i.e. Stall On All Writes

$$\text{Stall Cycles Per Memory Access} = \% \text{ reads} \times (1 - H1) \times M + \% \text{ write} \times M$$

$$\text{AMAT} = 1 + \% \text{ reads} \times (1 - H1) \times M + \% \text{ write} \times M$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access}$$

$$\text{Stall Cycles per access} = \text{AMAT} - 1$$

M = Miss Penalty
H1 = Level 1 Hit Rate
1 - H1 = Level 1 Miss Rate

M = Miss Penalty = stall cycles per access resulting from missing in cache
M + 1 = Miss Time = Main memory access time
H1 = Level 1 Hit Rate 1 - H1 = Level 1 Miss Rate

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Reducing Write Stalls For Write Through Cache Using Write Buffers

- To reduce write stalls when write through is used, a write buffer is used to eliminate or reduce write stalls:

- Perfect write buffer: All writes are handled by write buffer, no stalling for writes at all.

- In this case (for unified L1 cache):

$$\text{Stall Cycles Per Memory Access} = \% \text{ reads} \times (1 - H1) \times M$$

(i.e. No stalls at all for writes – Assuming buffer writes take 1 CPU cycle)

- Realistic Write buffer: A percentage of write stalls are not eliminated when the write buffer is full.

- In this case (for unified L1 cache):

$$\text{Stall Cycles/Memory Access} = (\% \text{ reads} \times (1 - H1) + \% \text{ write stalls not eliminated}) \times M$$

Write Through Cache Performance Example

- A CPU with $CPI_{\text{execution}} = 1.1$ Mem accesses per instruction = 1.3
- Uses a unified L1 Write Through, No Write Allocate, with:

1 – No write buffer.

2 – Perfect Write buffer

3 – A realistic write buffer that eliminates 85% of write stalls

- Instruction mix: 50% arith/logic, 15% load, 15% store, 20% control

- Assume a cache miss rate of 1.5% and a miss penalty of 50 cycles. = M

$$CPI = CPI_{\text{execution}} + \text{mem stalls per instruction}$$

$$\% \text{ reads} = 1.15/1.3 = 88.5\% \quad \% \text{ writes} = .15/1.3 = 11.5\%$$

1 With No Write Buffer : Stall on all writes

$$\text{Mem Stalls/ instruction} = 1.3 \times 50 \times (88.5\% \times 1.5\% + 11.5\%) = 8.33 \text{ cycles}$$

$$CPI = 1.1 + 8.33 = 9.43$$

2 With Perfect Write Buffer (all write stalls eliminated):

$$\text{Mem Stalls/ instruction} = 1.3 \times 50 \times (88.5\% \times 1.5\%) = 0.86 \text{ cycles}$$

$$CPI = 1.1 + 0.86 = 1.96$$

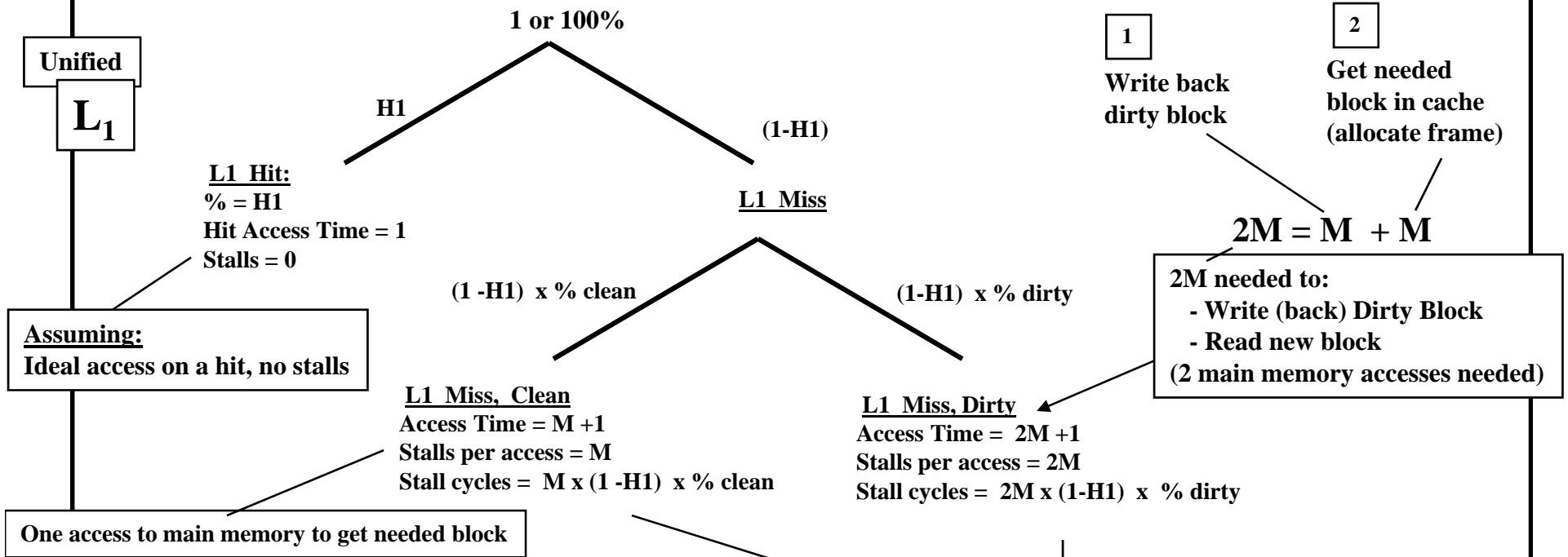
3 With Realistic Write Buffer (eliminates 85% of write stalls)

$$\text{Mem Stalls/ instruction} = 1.3 \times 50 \times (88.5\% \times 1.5\% + 15\% \times 11.5\%) = 1.98 \text{ cycles}$$

$$CPI = 1.1 + 1.98 = 3.08$$

Memory Access Tree Unified L₁ Write Back, With Write Allocate

CPU Memory Access



$$\text{Stall Cycles Per Memory Access} = (1-H1) \times (M \times \% \text{ clean} + 2M \times \% \text{ dirty})$$

$$\text{AMAT} = 1 + \text{Stall Cycles Per Memory Access}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access}$$

M = Miss Penalty = stall cycles per access resulting from missing in cache
M + 1 = Miss Time = Main memory access time
H1 = Level 1 Hit Rate 1 - H1 = Level 1 Miss Rate

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Write Back Cache Performance Example

- A CPU with $CPI_{\text{execution}} = 1.1$ uses a unified L1 with write back, with write allocate, and the probability a cache block is dirty = 10%
- Instruction mix: 50% arith/logic, 15% load, 15% store, 20% control
- Assume a cache miss rate of 1.5% and a miss penalty of 50 cycles.

$$CPI = CPI_{\text{execution}}^{(1-H1)} + \text{mem stalls per instruction}^{=M}$$

Mem Stalls per instruction =

Mem accesses per instruction x Stalls per access

$$\text{Mem accesses per instruction} = 1 + 0.3 = 1.3$$

$$\text{Stalls per access} = (1-H1) \times (M \times \% \text{ clean} + 2M \times \% \text{ dirty})$$

$$\text{Stalls per access} = 1.5\% \times (50 \times 90\% + 100 \times 10\%) = 0.825 \text{ cycles}$$

$$AMAT = 1 + \text{stalls per access} = 1 + 0.825 = 1.825 \text{ cycles}$$

$$\text{Mem Stalls per instruction} = 1.3 \times 0.825 = 1.07 \text{ cycles}$$

100 = 50 x 2
(For Dirty Misses)

$$CPI = 1.1 + 1.07 = 2.17$$

The ideal CPU with no misses is $2.17/1.1 = 1.97$ times faster

Assuming no stalls on a cache hit

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For Last Example

Memory Access Tree For Unified L₁ Write Back, With Write Allocate Example

CPU Memory Access

Given Parameters:

H1 = 98.5% T1 = 0 cycles

M = 50 cycles

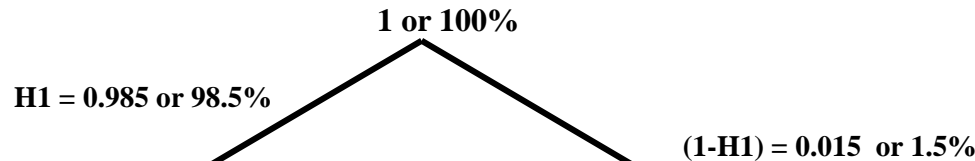
Stalls on a hit in L1

L1 Misses: 10% dirty 90% clean

CPI_{execution} = 1.1

Memory accesses per instruction = 1.3

Unified
L₁



L1 Hit:
 % = H1 = 0.985 or 98.5%
 Hit Access Time = 1
 Stalls = 0

L1 Miss

(1 - H1) x % clean
 = .015 x 0.9
 = 0.0135 or 1.35%

(1 - H1) x % dirty
 = 0.015 x 0.1
 = 0.0015 or 0.15%

Assuming:
Ideal access on a hit in L₁

2M needed to Write Dirty Block and Read new block

L1 Miss, Clean
 Access Time = M + 1 = 51
 Stalls per access = M = 50
 Stall cycles = M x (1 - H1) x % clean
 = 50 x 0.0135 = 0.675 cycles

L1 Miss, Dirty
 Access Time = 2M + 1 = 101
 Stalls per access = 2M = 100
 Stall cycles = 2M x (1 - H1) x % dirty
 = 100 x 0.0015 = 0.15 cycles

$$\text{Stall Cycles Per Memory Access} = M \times (1-H1) \times \% \text{ clean} + 2M \times (1-H1) \times \% \text{ dirty} = 0.675 + 0.15 = 0.825 \text{ cycles}$$

$$\text{AMAT} = 1 + \text{Stall Cycles Per Memory Access} = 1 + 0.825 = 1.825 \text{ cycles}$$

$$\text{Stall cycles per instruction} = (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access} = 1.3 \times 0.825 = 1.07 \text{ cycles}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + \text{Stall cycles per instruction} = 1.1 + 1.07 = 2.17$$

Given as 1.1

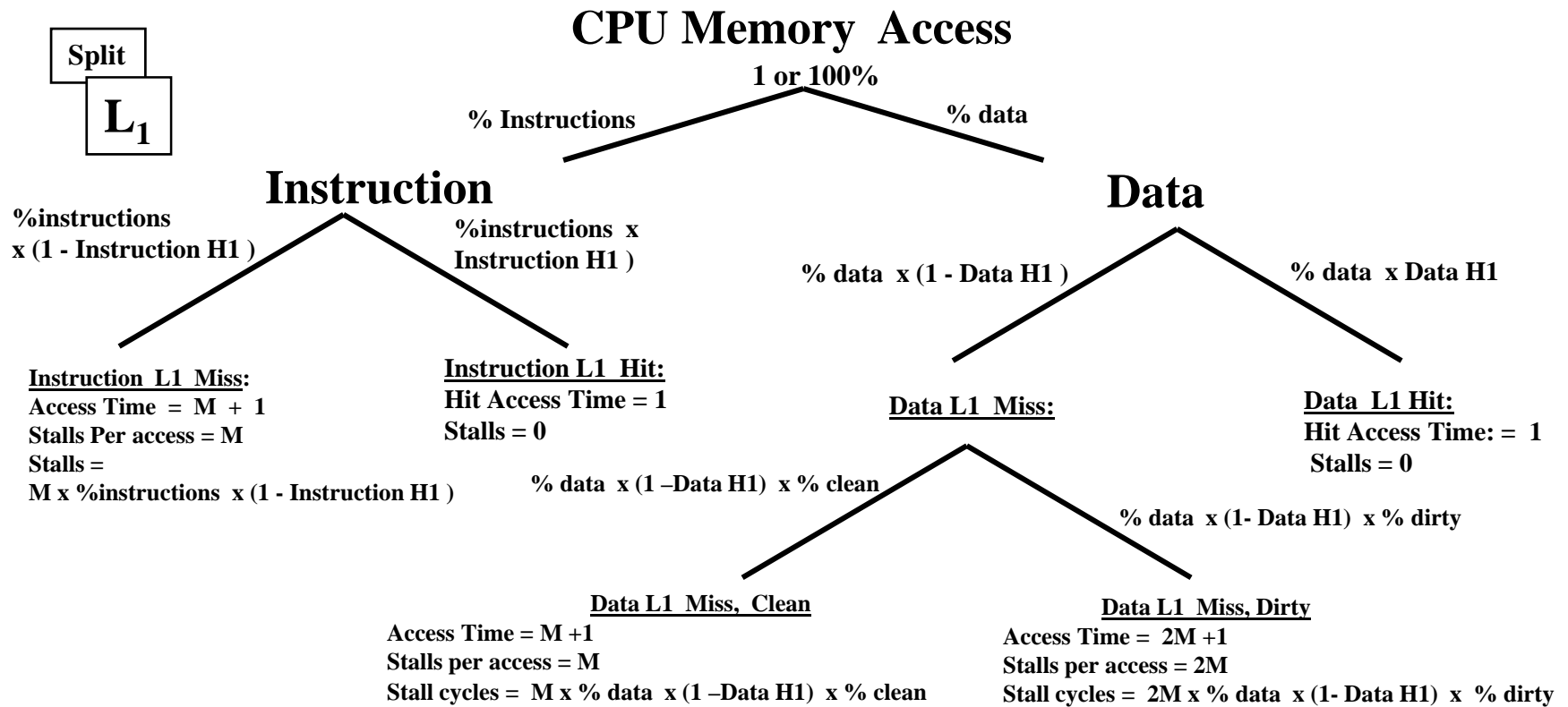
M = Miss Penalty = 50 cycles
 M + 1 = Miss Time = 50 + 1 = 51 cycles L1 access Time = 1 cycle
 H1 = 0.985 or 98.5% 1 - H1 = 0.015 or 1.5%

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Memory Access Tree Structure

For Separate Level 1 Caches, Write Back, With Write Allocate

(AKA Split)



Exercise: Find expression for: Stall cycles per average memory access, AMAT

M = Miss Penalty = stall cycles per access resulting from missing in cache
 M + 1 = Miss Time = Main memory access time
 Data H1 = Level 1 Data Hit Rate 1 - Data H1 = Level 1 Data Miss Rate
 Instruction H1 = Level 1 Instruction Hit Rate 1 - Instruction H1 = Level 1 Instruction Miss Rate
 % Instructions = Percentage or fraction of instruction fetches out of all memory accesses
 % Data = Percentage or fraction of data accesses out of all memory accesses
 % Clean = Percentage or fraction of data L1 misses that are clean
 % Dirty = Percentage or fraction of data L1 misses that are dirty = 1 - % Clean

Assuming:
 Ideal access on a hit in L₁

Improving Cache Performance: Multi-Level Cache

2 Levels of Cache: L_1, L_2

Basic Design Rule for L_1 Cache:

K.I.S.S

(e.g low degree of associativity and capacity to keep it fast)

3-4 cycles access time
2-8 way set associative

CPU

L_1 Cache

64k + 64K ?

Hit Rate = H_1

Hit Access Time = 1 cycle (No Stall)

Stalls for hit access = $T_1 = 0$

Assuming
Ideal access on a hit in L_1

L_2 has slower access time than L_1 , but has more capacity and higher associativity

10-15 cycles access time
8-16 way set associative

L_2 Cache

256k- 6MB ?

Local Hit Rate = H_2

Stalls per hit access = T_2

Hit Access Time = $T_2 + 1$ cycles

Local ?

Main Memory

200+ Cycles ?

Slower (longer access time) than L_2

4- 16 GB ?

Memory access penalty, M

(stalls per main memory access)

Access Time = $M + 1$

Typically 200+ Cycles

L_1 = Level 1 Cache

L_2 = Level 2 Cache

Goal of multi-level Caches:

Reduce the effective miss penalty incurred by level 1 cache misses by using additional levels of cache that capture some of these misses.

Thus hiding more main memory latency and reducing AMAT further

4th Edition: Appendix C.3
(3rd Edition Chapter 5.4)

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Miss Rates For Multi-Level Caches

i.e that reach this level

- **Local Miss Rate:** This rate is the number of misses in a cache level divided by the number of memory accesses to this level (i.e those memory accesses that reach this level).

$$\text{Local Hit Rate} = 1 - \text{Local Miss Rate}$$

- **Global Miss Rate:** The number of misses in a cache level divided by the total number of memory accesses generated by the CPU.

- L_1 • Since level 1 receives all CPU memory accesses, for level 1:

$$\text{Local Miss Rate} = \text{Global Miss Rate} = 1 - H1$$

- L_2 • For level 2 since it only receives those accesses missed in 1:

$$\text{Local Miss Rate} = \text{Miss rate}_{L_2} = 1 - H2 \quad \text{--- H2 Local Hit Rate for L2}$$

$$\begin{aligned} \text{Global Miss Rate} &= \text{Miss rate}_{L_1} \times \text{Local Miss rate}_{L_2} \\ &= (1 - H1) \times (1 - H2) \end{aligned}$$

For Cache Level 3 (L_3), global miss rate?

Local? Global?

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2-Level Cache (Both Unified) Performance (Ignoring Write Policy)

$$\text{CPUtime} = \text{IC} \times (\text{CPI}_{\text{execution}} + \text{Mem Stall cycles per instruction}) \times C$$

$$\text{Mem Stall cycles per instruction} = \text{Mem accesses per instruction} \times \text{Stall cycles per access}$$

- For a system with 2 levels of unified cache, assuming no penalty when found in L₁ cache: (T₁ = 0)

$$\text{AMAT} = 1 + \text{Stall Cycles per access}$$

Average stall cycles per memory access =

$$[\text{miss rate } L_1] \times [\text{Local Hit rate } L_2 \times \text{Hit time } L_2 + \text{Local Miss rate } L_2 \times \text{Memory access penalty}] =$$

L1 Hit (here we assume T₁ = 0)

$$H1 \times 0 +$$

$$(1-H1) \times H2 \times T2 + (1-H1)(1-H2) \times M$$

L1 Miss, L2 Hit

Here we assume T₁ = 0
(no stall on L1 hit)

H1 = L1 Hit Rate

T₁ = stall cycles per L1 access hit

H2 = Local L2 Hit Rate

T₂ = stall cycles per L2 access hit

L1 Miss, L2 Miss:
Must Access Main Memory

Full Miss

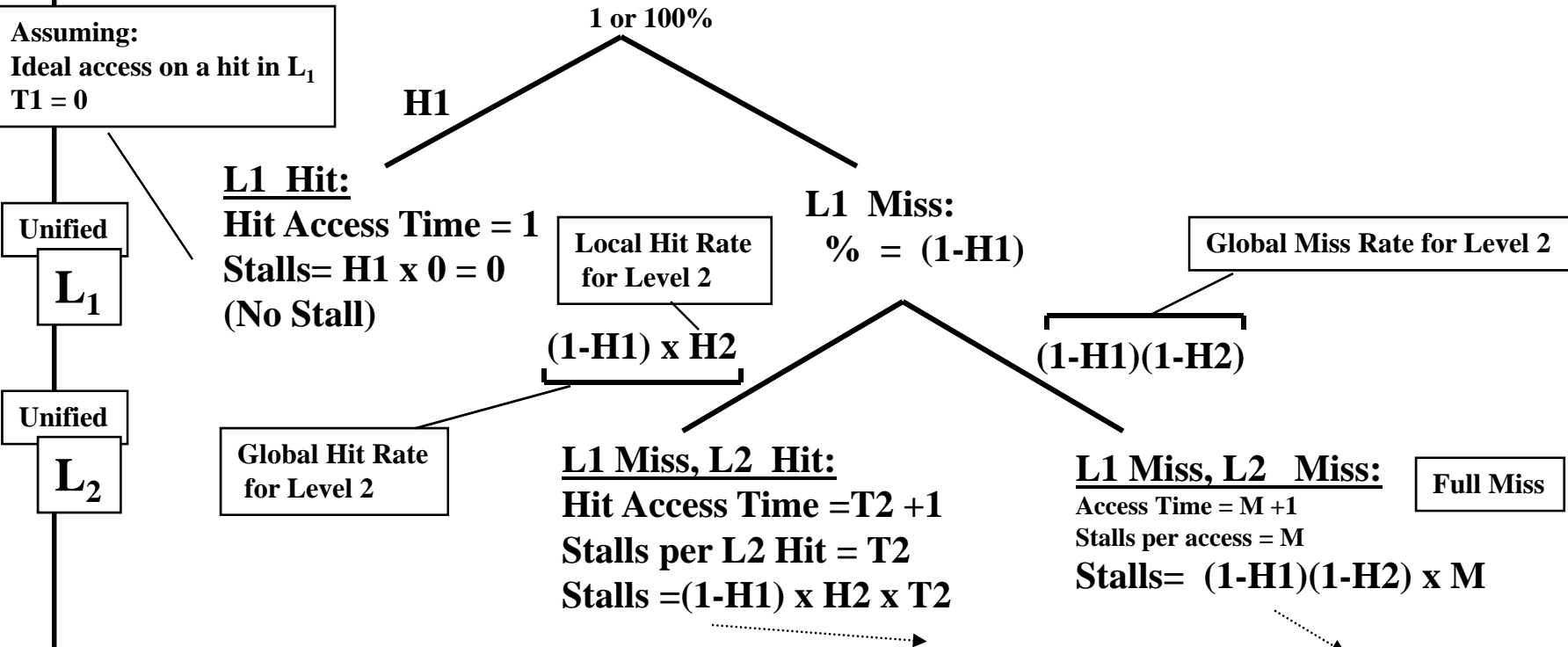
$$\begin{aligned} \text{CPI} &= \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access} \\ &= \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (\text{AMAT} - 1) \end{aligned}$$

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2-Level Cache (Both Unified) Performance Memory Access Tree (Ignoring Write Policy)

CPU Stall Cycles Per Memory Access

CPU Memory Access



$$\text{Stall cycles per memory access} = (1-H1) \times H2 \times T2 + (1-H1)(1-H2) \times M$$

$$\text{AMAT} = 1 + (1-H1) \times H2 \times T2 + (1-H1)(1-H2) \times M$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access}$$

$$= \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (\text{AMAT} - 1)$$

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Unified Two-Level Cache Example

- CPU with $CPI_{\text{execution}} = 1.1$ running at clock rate = 500 MHz (Ignoring Write Policy)
- 1.3 memory accesses per instruction.
- With two levels of cache (both unified)
- L_1 hit access time = 1 cycle (no stall on a hit, $T_1 = 0$), a miss rate of 5%
- L_2 hit access time = 3 cycles ($T_2 = 2$ stall cycles per hit) with local miss rate 40%,
- Memory access penalty, $M = 100$ cycles (stalls per access). **Find CPI ...**

$$CPI = CPI_{\text{execution}} + \text{Mem Stall cycles per instruction}$$

$$\text{With No Cache, } CPI = 1.1 + 1.3 \times 100 = 131.1$$

$$\text{With single } L_1, \text{ } CPI = 1.1 + 1.3 \times .05 \times 100 = 7.6$$

$$\text{Mem Stall cycles per instruction} = \text{Mem accesses per instruction} \times \text{Stall cycles per access}$$

$$\text{Stall cycles per memory access} = (1-H_1) \times H_2 \times T_2 + (1-H_1)(1-H_2) \times M$$

$$= 0.05 \times .6 \times 2 + 0.05 \times 0.4 \times 100 \quad \text{+ H1 x 0}$$

$$= 0.06 + 2 = 2.06 \text{ cycles}$$

$$AMAT = 2.06 + 1 = 3.06 \text{ cycles}$$

$$\text{Mem Stall cycles per instruction} = \text{Mem accesses per instruction} \times \text{Stall cycles per access}$$

$$= 2.06 \times 1.3 = 2.678 \text{ cycles}$$

$$CPI = 1.1 + 2.678 = 3.778$$

$$\text{Speedup} = 7.6/3.778 = 2 \quad \text{Compared to CPU with L1 only}$$

i.e 1-H2

L1 Hit (here we assume $T_1 = 0$)

$$CPI = CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access}$$

$$= CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (AMAT - 1)$$

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Memory Access Tree For 2-Level Cache (Both Unified) Example

For Last Example

(Ignoring Write Policy)

CPU Stall Cycles Per Memory Access

CPU Memory Access

1 or 100%

$H1 = 0.95$ or 95%

Unified

L₁

L1 Hit:

Hit Access Time = 1
 Stalls per L1 Hit = $T1 = 0$
 Stalls = $H1 \times 0 = 0$
 (No Stall)

L1 Miss:

$(1-H1) = 0.05$ or 5%

Given Parameters:

$H1 = 95\%$ $T1 = 0$ cycles

$H2 = 60\%$ $T2 = 2$ cycles

$M = 100$ cycles

Stalls on a hit

$CPI_{\text{execution}} = 1.1$

Memory accesses per instruction = 1.3

Ideal access on a hit in L₁
 $T1 = 0$

$(1-H1) \times H2$
 $= 0.05 \times 0.6$
 $= 0.03$ or 3%

$(1-H1)(1-H2)$
 $= 0.05 \times 0.4$
 $= 0.02$ or 2%

Global Miss Rate for L₂

Unified

L₂

Global Hit Rate for L₂

L1 Miss, L2 Hit:

Hit Access Time = $T2 + 1 = 3$ cycles
 Stalls per L2 Hit = $T2 = 2$ cycles
 Stalls = $(1-H1) \times H2 \times T2$
 $= 0.03 \times 2 = 0.06$ cycles

L1 Miss, L2 Miss:

Access Time = $M + 1 = 100 + 1 = 101$ cycles
 Stalls per access = $M = 100$ cycles
 Stalls = $(1-H1)(1-H2) \times M$
 $= 0.02 \times 100 = 2$ cycles

Full Miss

$$\begin{aligned} \text{Stall cycles per memory access} &= (1-H1) \times H2 \times T2 + (1-H1)(1-H2) \times M \\ &= 0.06 + 2 = 2.06 \text{ cycles} \end{aligned}$$

$$AMAT = 1 + \text{Stall cycles per memory access} = 1 + 2.06 = 3.06 \text{ cycles}$$

$$\begin{aligned} \text{Stall cycles per instruction} &= (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access} \\ &= 1.3 \times 2.06 = 2.678 \text{ cycles} \end{aligned}$$

$$CPI = CPI_{\text{execution}} + \text{Stall cycles per instruction} = 1.1 + 2.678 = 3.778$$

$$\begin{aligned} CPI &= CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access} \\ &= CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (AMAT - 1) \end{aligned}$$

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Memory Access Tree Structure For 2-Level Cache (Separate Level 1 Caches, Unified Level 2)

(Ignoring Write Policy)

CPU Memory Access

1 or 100%

% Instructions % data

Split
L₁

Instruction

Data

Instruction L1 Hit:

Instruction L1 Miss:

Data L1 Hit:

Data L1 Miss:

Unified
L₂

L2 Hit

L2 Miss

L2 Hit

L2 Miss

Exercise: In terms of the parameters below, complete the memory access tree and find the expression for stall cycles per memory access

% Instructions = Percentage or fraction of instruction fetches out of all memory accesses
% Data = Percentage or fraction of data accesses out of all memory accesses

For L1: T1 = Stalls per hit access to level 1
Data H1 = Level 1 Data Hit Rate 1- Data H1 = Level 1 Data Miss Rate
Instruction H1 = Level 1 Instruction Hit Rate 1- Instruction H1 = Level 1 Instruction Miss Rate

For L2: T2 = Stalls per access to level 2
H2 = Level 2 local hit Rate 1-H2 = Level 2 local miss rate

M = Miss Penalty = stall cycles per access resulting from missing in cache level 2
M + 1 = Miss Time = Main memory access time

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Common Write Policy For 2-Level Cache

- L₁**
- **Write Policy For Level 1 Cache:**
 - Usually **Write through to Level 2.** (not write through to main memory just to L2)
 - **Write allocate** is used to reduce level 1 read misses.
 - Use **write buffer** to reduce write stalls to level 2.

- L₂**
- **Write Policy For Level 2 Cache:**
 - Usually **write back with write allocate** is used.
 - To minimize memory bandwidth usage.



- The above 2-level cache write policy results in **inclusive L2 cache** since the content of L1 is also in L2
 - **Common in the majority of all CPUs with 2-levels of cache**
 - As opposed to exclusive L1, L2 (e.g AMD Athlon XP, A64)

As if we have a single level of cache with one portion (L1) is faster than remainder (L2)

i.e what is in L1 is not duplicated in L2



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2-Level (Both Unified) Memory Access Tree

L1: Write Through to L2, Write Allocate, With Perfect Write Buffer

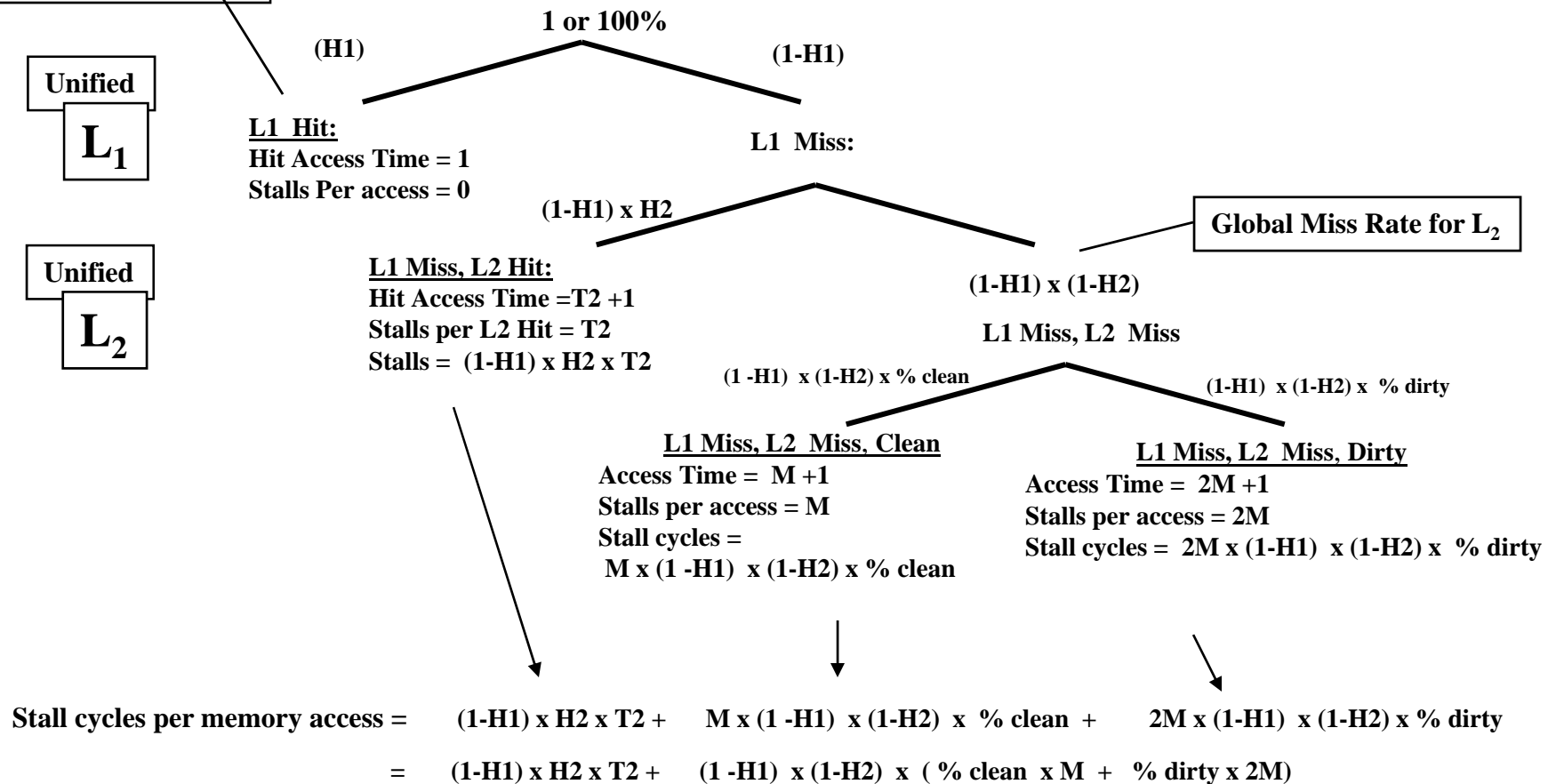
L2: Write Back with Write Allocate

Assuming:

Ideal access on a hit in L₁

T₁ = 0

CPU Memory Access



AMAT = 1 + Stall Cycles Per Memory Access

CPI = CPI_{execution} + (1 + fraction of loads and stores) x Stall Cycles per access

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Two-Level (Both Unified) Cache Example With Write Policy

- CPU with $CPI_{\text{execution}} = 1.1$ running at clock rate = 500 MHz
- 1.3 memory accesses per instruction. Two levels of cache (both unified)
- **For L_1 :** i.e. ideal access time = 1 cycle
 - Cache operates at 500 MHz (no stall on L1 Hit, $T_1 = 0$) with a miss rate of $1 - H_1 = 5\%$
 - Write through to L_2 with perfect write buffer with write allocate
- **For L_2 :**
 - Hit access time = 3 cycles ($T_2 = 2$ stall cycles per hit) local miss rate $1 - H_2 = 40\%$
 - Write back to main memory with write allocate
 - Probability a cache block is dirty = 10%
- Memory access penalty, $M = 100$ cycles.
- Create memory access tree and find, stalls per memory access, AMAT, CPI.
- Stall cycles per memory access = $(1 - H_1) \times H_2 \times T_2 + (1 - H_1) \times (1 - H_2) \times (\% \text{ clean} \times M + \% \text{ dirty} \times 2M)$
 $= .05 \times .6 \times 2 + .05 \times .4 \times (.9 \times 100 + .1 \times 200)$
 $= .06 + 0.02 \times 110 = .06 + 2.2 = 2.26$
- $AMAT = 2.26 + 1 = 3.26$ cycles

$$\text{Mem Stall cycles per instruction} = \text{Mem accesses per instruction} \times \text{Stall cycles per access}$$

$$= 2.26 \times 1.3 = 2.938 \text{ cycles}$$

$$CPI = 1.1 + 2.938 = 4.038 = 4$$

$$CPI = CPI_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (AMAT - 1)$$

Memory Access Tree For Two-Level (Both Unified) Cache Example With Write Policy

L1: Write Through to L2, Write Allocate, With Perfect Write Buffer

L2: Write Back with Write Allocate

For Last Example

Given Parameters:

H1 = 95% T1 = 0 cycles

H2 = 60% T2 = 2 cycles

M = 100 cycles

Stalls on a hit

L2 Misses: 10% dirty 90% clean

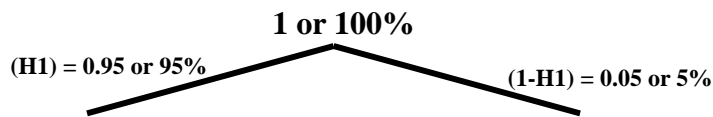
CPI_{execution} = 1.1

Memory accesses per instruction = 1.3

CPU Memory Access

Unified

L₁



L1 Hit:

Hit Access Time = 1
Stalls Per access = 0

L1 Miss:

(1-H1) x H2
= 0.05 x 0.6
= 0.03 or 3%

Unified

L₂

L1 Miss, L2 Hit:

Hit Access Time = T2 + 1 = 3 cycles
Stalls per L2 Hit = T2 = 2 cycles
Stalls = (1-H1) x H2 x T2
= 0.03 x 2 = 0.06 cycles

(1-H1) x (1-H2) = 0.05 x 0.4 = 0.02 or 2%

L1 Miss, L2 Miss

(1-H1) x (1-H2) x % clean
= 0.02 x 0.9 = 0.018 or 1.8%

(1-H1) x (1-H2) x % dirty
= 0.02 x 0.1 = 0.002 or 0.2 %

L1 Miss, L2 Miss, Clean

Access Time = M + 1 = 101 cycles
Stalls per access = M
Stall cycles = M x (1-H1) x (1-H2) x % clean
= 100 x 0.018 = 1.8 cycles

L1 Miss, L2 Miss, Dirty

Access Time = 2M + 1 = 200 + 1 = 201 cycles
Stalls per access = 2M = 200 cycles
Stall cycles = 2M x (1-H1) x (1-H2) x % dirty
= 200 x 0.002 = 0.4 cycles

$$\text{Stall cycles per memory access} = (1-H1) \times H2 \times T2 + M \times (1-H1) \times (1-H2) \times \% \text{ clean} + 2M \times (1-H1) \times (1-H2) \times \% \text{ dirty}$$

$$= 0.06 + 1.8 + 0.4 = 2.26 \text{ cycles}$$

$$\text{AMAT} = 1 + \text{Stall cycles per memory access} = 1 + 2.26 = 3.26 \text{ cycles}$$

$$\text{Stall cycles per instruction} = (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access}$$

$$= 1.3 \times 2.26 = 2.938 \text{ cycles}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + \text{Stall cycles per instruction} = 1.1 + 2.938 = 4.038$$

$$\text{AMAT} = 1 + \text{Stall Cycles Per Memory Access}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{Stall Cycles per access}$$

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Memory Access Tree Structure For 2-Level Cache (Separate Level 1 Caches, Unified Level 2)
L1: Write Through to L2, Write Allocate, With Perfect Write Buffer **L2: Write Back with Write Allocate**

CPU Memory Access

1 or 100%

% Instructions

% data

Instruction

Data

Split
L₁

Instruction L1 Hit:

Instruction L1 Miss:

Data L1 Hit:

Data L1 Miss:

Unified
L₂

L2 Hit

L2 Miss

L2 Hit

L2 Miss

L2 Miss Clean

L2 Miss Dirty

L2 Miss Clean

L2 Miss Dirty

Exercise: In terms of the parameters below, complete the memory access tree and find the expression for stall cycles per memory access

% Instructions = Percentage or fraction of instruction fetches out of all memory accesses
 % Data = Percentage or fraction of data accesses out of all memory accesses

For L1: T1 = Stalls per hit access to level 1

Data H1 = Level 1 Data Hit Rate

1- Data H1 = Level 1 Data Miss Rate

Instruction H1 = Level 1 Instruction Hit Rate

1- Instruction H1 = Level 1 Instruction Miss Rate

For L2: T2 = Stalls per access to level 2

H2 = Level 2 local hit Rate 1-H2 = Level 2 local miss rate

% Clean = Percentage or fraction of data L2 misses that are clean

% Dirty = Percentage or fraction of L2 misses that are dirty = 1 - % Clean

M = Miss Penalty = stall cycles per access resulting from missing in cache level 2

M + 1 = Miss Time = Main memory access time

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Multi-Level Cache:

3 Levels of Cache

Basic Design Rule for L₁ Cache:

K.I.S.S

(e.g low degree of associativity and capacity to keep it fast)

3-4 cycles access time
2-8 way set associative

Slower than L₁
But has more capacity and higher associativity

10-15 cycles access time
8-16 way set associative

Slower the L₂
But has more capacity and higher associativity

30-60 cycles access time
16-64 way set associative

CPU

L1 Cache

64k + 64K ?

L2 Cache

256k- 6MB ?

L3 Cache

Up to 24MB ?

Main Memory

200+ Cycles ?

Slower the L₃

4- 16 GB ?

Memory access penalty, M

(stalls per main memory access)

Access Time = M + 1

Typically 200+ Cycles

L₁ = Level 1 Cache

L₂ = Level 2 Cache

L₃ = Level 3 Cache

Assuming
Ideal access on a hit in L₁

Hit Rate= H₁,

Hit Access Time = 1 cycle (No Stall)

Stalls for hit access = T₁ = 0

Local Hit Rate= H₂

Stalls per hit access= T₂

Hit Access Time = T₂ + 1 cycles

Local Hit Rate= H₃

Stalls per hit access= T₃

Hit Access Time = T₃ + 1 cycles

$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access}$$

$$= \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (\text{AMAT} - 1)$$

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3-Level (All Unified) Cache Performance

(Ignoring Write Policy)

$$\text{CPUtime} = \text{IC} \times (\text{CPI}_{\text{execution}} + \text{Mem Stall cycles per instruction}) \times C$$

$$\text{Mem Stall cycles per instruction} = \text{Mem accesses per instruction} \times \text{Stall cycles per access}$$

- For a system with 3 levels of cache, assuming no penalty when found in L₁ cache: (T₁ = 0)

Stall cycles per memory access =

$$[\text{miss rate } L_1] \times [\text{Hit rate } L_2 \times \text{Hit time } L_2 + \text{Miss rate } L_2 \times (\text{Hit rate } L_3 \times \text{Hit time } L_3 + \text{Miss rate } L_3 \times \text{Memory access penalty})] =$$

L1 Hit (here we assume T1 = 0)

H1 x 0 +

$$(1-H1) \times H2 \times T2$$

L1 Miss, L2 Hit

$$+ (1-H1) \times (1-H2) \times H3 \times T3$$

L1 Miss, L2 Miss, L3 Hit

$$+ (1-H1)(1-H2) (1-H3) \times M$$

L1 Miss, L2 Miss, L3 Miss: Must Access Main Memory

Full Miss

$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access}$$

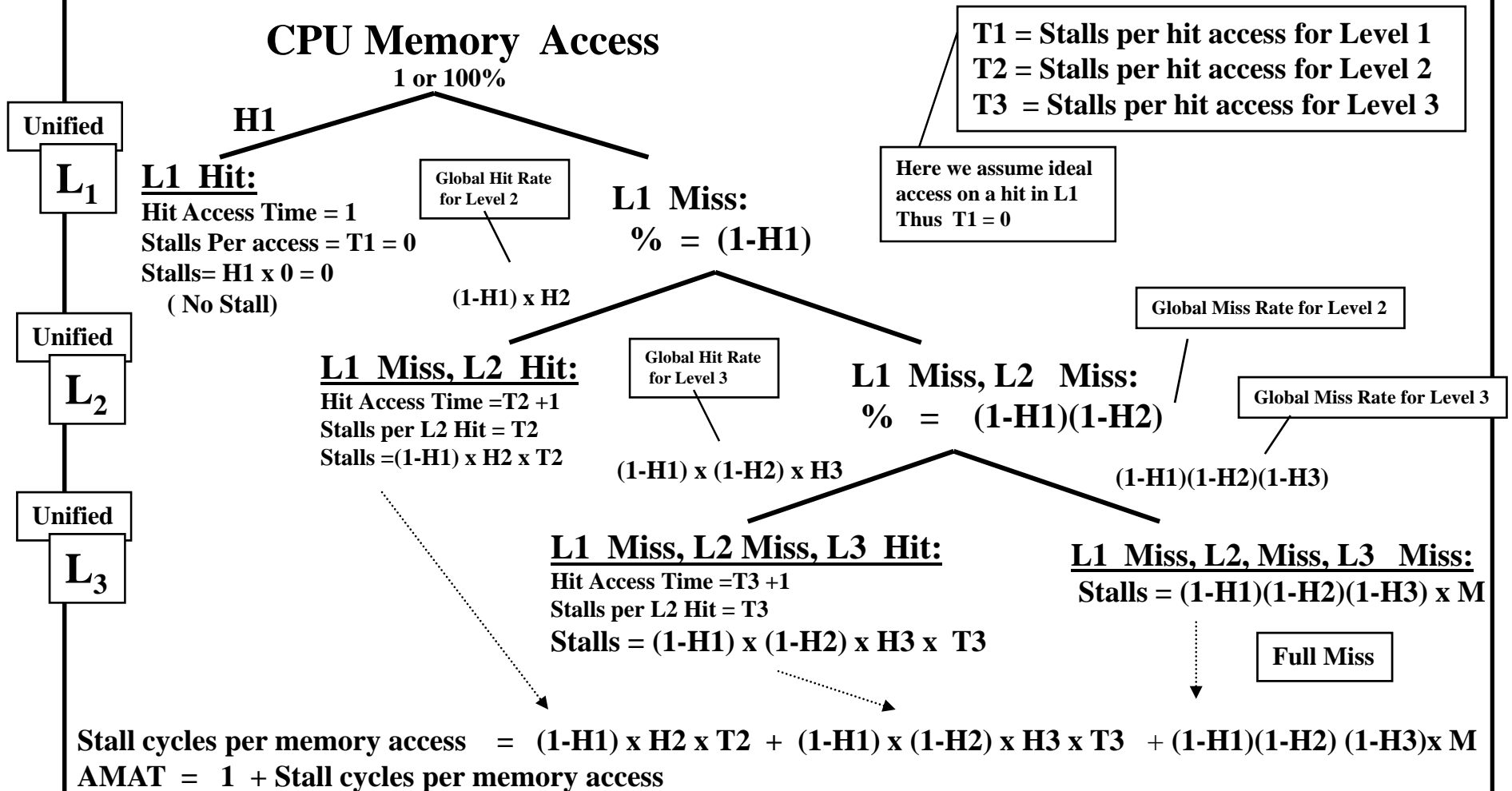
$$= \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (\text{AMAT} - 1)$$

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3-Level (All Unified) Cache Performance

Memory Access Tree (Ignoring Write Policy)

CPU Stall Cycles Per Memory Access



$$\text{CPI} = \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times \text{stall cycles per access}$$

$$= \text{CPI}_{\text{execution}} + (1 + \text{fraction of loads and stores}) \times (\text{AMAT} - 1)$$

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Three-Level (All Unified) Cache Example

- CPU with $CPI_{\text{execution}} = 1.1$ running at clock rate = 500 MHz
- 1.3 memory accesses per instruction.
- L_1 cache operates at 500 MHz (no stalls on a hit in L_1) with a miss rate of 5%
- L_2 hit access time = 3 cycles ($T_2 = 2$ stall cycles per hit), local miss rate 40%
- L_3 hit access time = 6 cycles ($T_3 = 5$ stall cycles per hit), local miss rate 50%,
- Memory access penalty, $M = 100$ cycles (stall cycles per access). Find CPI.

(Ignoring Write Policy)

With No Cache, $CPI = 1.1 + 1.3 \times 100 = 131.1$

With single L_1 , $CPI = 1.1 + 1.3 \times .05 \times 100 = 7.6$

With L_1, L_2 $CPI = 1.1 + 1.3 \times (.05 \times .6 \times 2 + .05 \times .4 \times 100) = 3.778$

$$CPI = CPI_{\text{execution}} + \text{Mem Stall cycles per instruction}$$

Mem Stall cycles per instruction = Mem accesses per instruction \times Stall cycles per access

$$\begin{aligned} \text{Stall cycles per memory access} &= (1-H_1) \times H_2 \times T_2 + (1-H_1) \times (1-H_2) \times H_3 \times T_3 + (1-H_1)(1-H_2)(1-H_3) \times M \\ &= .05 \times .6 \times 2 + .05 \times .4 \times .5 \times 5 + .05 \times .4 \times .5 \times 100 \\ &= .06 + .05 + 1 = 1.11 \end{aligned}$$

AMAT = 1.11 + 1 = 2.11 cycles (vs. AMAT = 3.06 with L_1, L_2 , vs. 5 with L_1 only)

$$CPI = 1.1 + 1.3 \times 1.11 = 2.54$$

Speedup compared to L_1 only = $7.6/2.54 = 3$

Speedup compared to L_1, L_2 = $3.778/2.54 = 1.49$

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Memory Access Tree For 3-Level Cache (All Unified) Example

For Last Example

CPU Memory Access

(Ignoring Write Policy)

H1 = .95 or 95%
1 or 100%

Unified

L₁

L1 Hit:

Hit Access Time = 1
Stalls Per access = 0
Stalls = H1 x 0 = 0
(No Stall)

Global Hit Rate
for Level 2

$$(1-H1) \times H2 = 0.05 \times .6 = 0.03 \text{ or } 3\%$$

L1 Miss:

(1-H1) = 0.05 or 5%

Given Parameters:

H1 = 95% T1 = 0 cycles
H2 = 60% T2 = 2 cycles
H3 = 50% T3 = 5 cycles

M = 100 cycles

Stalls on a hit

CPI_{execution} = 1.1

Memory accesses per instruction = 1.3

Unified

L₂

L1 Miss, L2 Hit:

Hit Access Time = T2 + 1 = 3
Stalls per L2 Hit = T2 = 2
Stalls = (1-H1) x H2 x T2 = .05 x .6 x 2 = .06

Global Hit Rate
for Level 3

$$(1-H1) \times (1-H2) \times H3 = .05 \times .4 \times .5 = .01 \text{ or } 1\%$$

L1 Miss, L2 Miss:

(1-H1)(1-H2) = .05 x .4 = .02 or 2%

Global Miss Rate for Level 2

$$(1-H1)(1-H2)(1-H3) = .05 \times .4 \times .5 = .01 \text{ or } 1\%$$

Global Miss Rate
for Level 3

Unified

L₃

L1 Miss, L2 Miss, L3 Hit:

Hit Access Time = T3 + 1 = 6
Stalls per L2 Hit = T3 = 5
Stalls = (1-H1) x (1-H2) x H3 x T3 = .01 x 5 = .05 cycles

L1 Miss, L2, Miss, L3 Miss:

Miss Penalty = M = 100
Stalls = (1-H1)(1-H2)(1-H3) x M = .01 x 100 = 1 cycle

Full Miss

$$\text{Stall cycles per memory access} = (1-H1) \times H2 \times T2 + (1-H1) \times (1-H2) \times H3 \times T3 + (1-H1)(1-H2)(1-H3) \times M = .06 + .05 + 1 = 1.11$$

$$\text{AMAT} = 1 + \text{Stall cycles per memory access} = 1 + 1.11 = 2.11 \text{ cycles}$$

$$\text{Stall cycles per instruction} = (1 + \text{fraction of loads/stores}) \times \text{Stall Cycles per access} = 1.3 \times 1.11 = 1.443 \text{ cycles}$$

$$\text{CPI} = \text{CPI}_{\text{execution}} + \text{Stall cycles per instruction} = 1.1 + 1.443 = 2.543$$

Exercise: Create the memory access tree for 3-level cache where level 1 is split and Levels 2, 3 are unified once ignoring write policy and once with write back for L3. Find the expression for memory stalls per access for either case.

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