Clocked Synchronous State-Machines

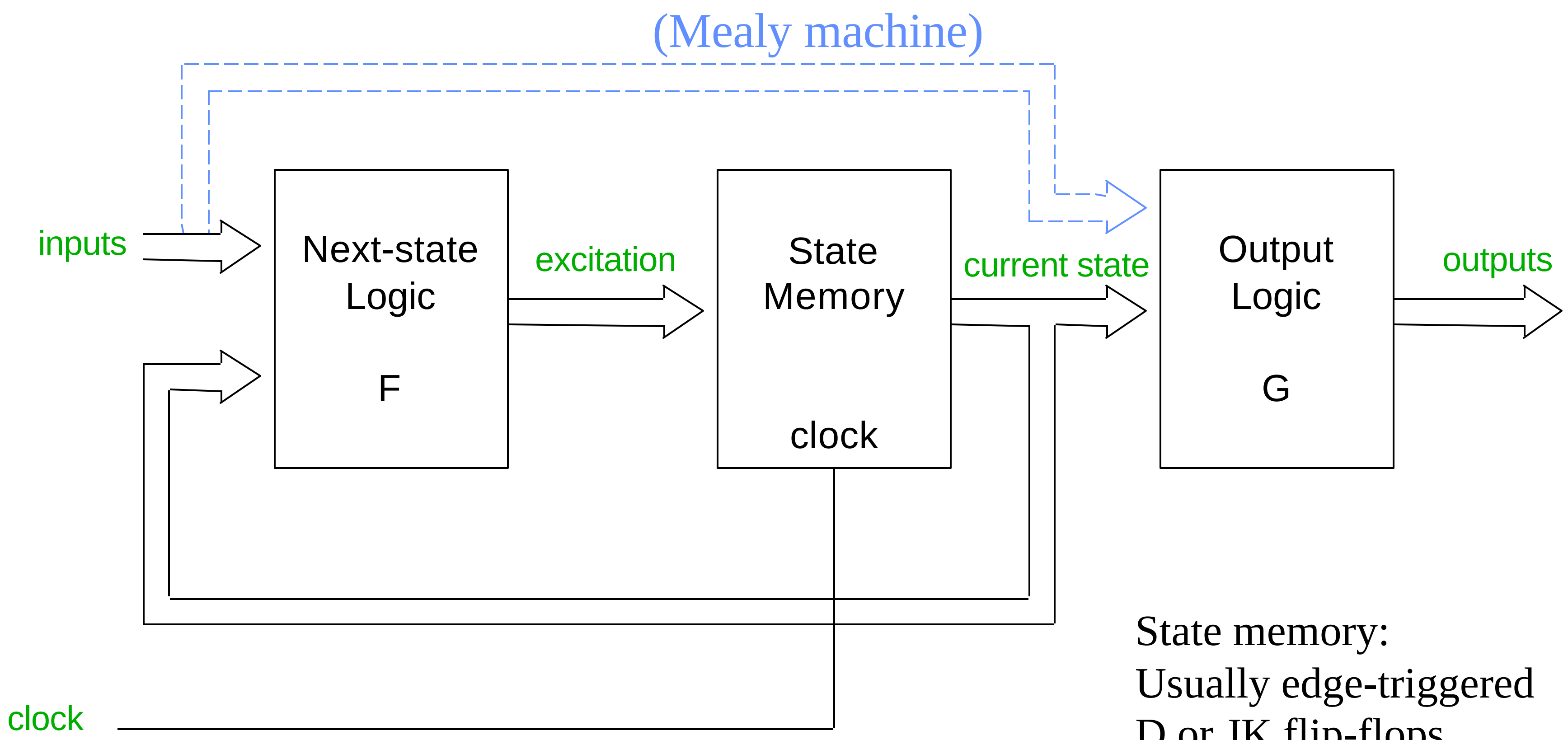
- Such machines have the characteristics:
  - Sequential circuits designed using flip-flops.
  - All flip-flops use a common clock (clocked synchronous).
  - A machine using n flip-flops (state memory) has n state variables (the outputs of the flip-flops) and 2^n states.
  - In general, the next state and output of the machine both depend on the current state of the machine and on the current input:

> Next state = F(current state, input)
>
> output = G(current state, input)
>
> This type of state machine is called Mealy Machine

  - In some cases the next output depends only on the current state and not directly on the current input

> Next state = F(current state, input)   output = G(current state)
>
> Such machines are called Moore machines.

Clocked Synchronous State-Machine Model



State memory:
Usually edge-triggered
D or JK flip-flops

Moore Machine

Latch/Flip-Flop Characteristic Equations

- **The next output of a flip flop (or next state) can be obtained from the function table of each type of flip-flop.**
- **This latch/flip-flop next output behavior is expressed in as a characteristic function which gives the next state in terms of the current state and output:**

$$Q^* = f\ (Q\ ,\ \text{inputs})$$

(Q^* is the next state of Q).

- **Vary important in state machine analysis and design.**

Characteristic Equations

D latch or flip-flop

input D	present state Q(t)	next state Q*
0	0	0
0	1	0
1	0	1
1	1	1

Characteristic Equation:

Q* = D

S-R latch

S	R	Q(t)	Q*
0	0	0	0
0	0	1	1
0	1	X	0
1	0	X	1
1	1	X	X

Characteristic Equation:

Q* = S + R’. Q

Characteristic Equations

J-K flip-flop

J	K	Q	Q*	
0	0	0	0	
0	0	1	1	= hold
0	1	0	0	
0	1	1	0	= reset
1	0	0	1	
1	0	1	1	= set
1	1	0	1	
1	1	1	0	= flip

Characteristic Equation:

Q* = J . Q' + K'. Q

T flip-flop with enable

T	Q	Q*
0	0	0
0	1	1
1	0	1
1	1	0

Characteristic Equation:

Q* = T. Q' + T' .Q

Latch/Flip-Flop Characteristic Equations

Device	Characteristic Equations
S-R latch	Q* = S+R'.Q
D latch	Q* = D
Edge-triggered D flip-flop	Q* = D
Master/Slave S-R flip-flop	Q* = S+R'.Q
Master/Slave J-K flip flop	Q* = J.Q' + K'.Q
Edge Triggered J-K flip-flop	Q* = J.Q' + K'.Q
T flip-flop	Q* = Q'
T flip-flop with enable	Q* = EN.Q' + EN'.Q

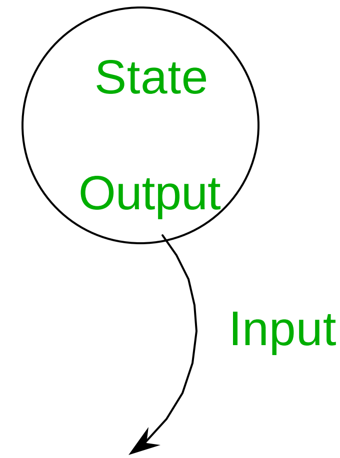
Clocked Synchronous State-machine Analysis

Given the circuit diagram of a state machine:

1. **Analyze the combinational logic to determine flip-flop input (excitation) equations:** $D_i = F_i\,(Q, \text{inputs})$
   - **The input to each flip-flop is based upon current state and circuit inputs.**
2. **Substitute excitation equations into flip-flop characteristic equations, giving transition equations:** $Q_i^* = H_i(D_i)$
3. **From the circuit, find output equations:** $Z = G\,(Q, \text{inputs})$
   - **The outputs are based upon the current state and possibly the inputs.**
4. **Construct a state transition/output table from the transition and output equations:**
   - **Similar to truth table.**
   - **Present state on the left side.**
   - **Outputs and next state for each input value on the right side.**
   - **Provide meaningful names for the states in state table, if possible.**
5. **Draw the state diagram which is the graphical representation of state table.**

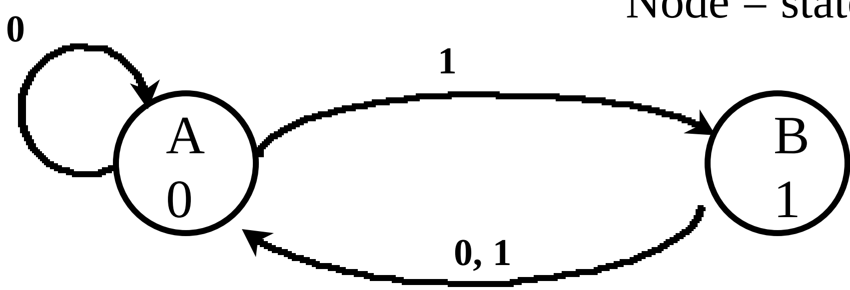
State Diagram

Basic Format:

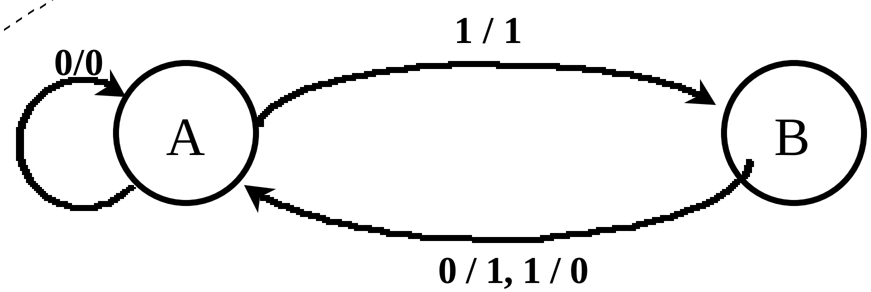


Moore

Format:
Arc = input X
Node = state/output Q



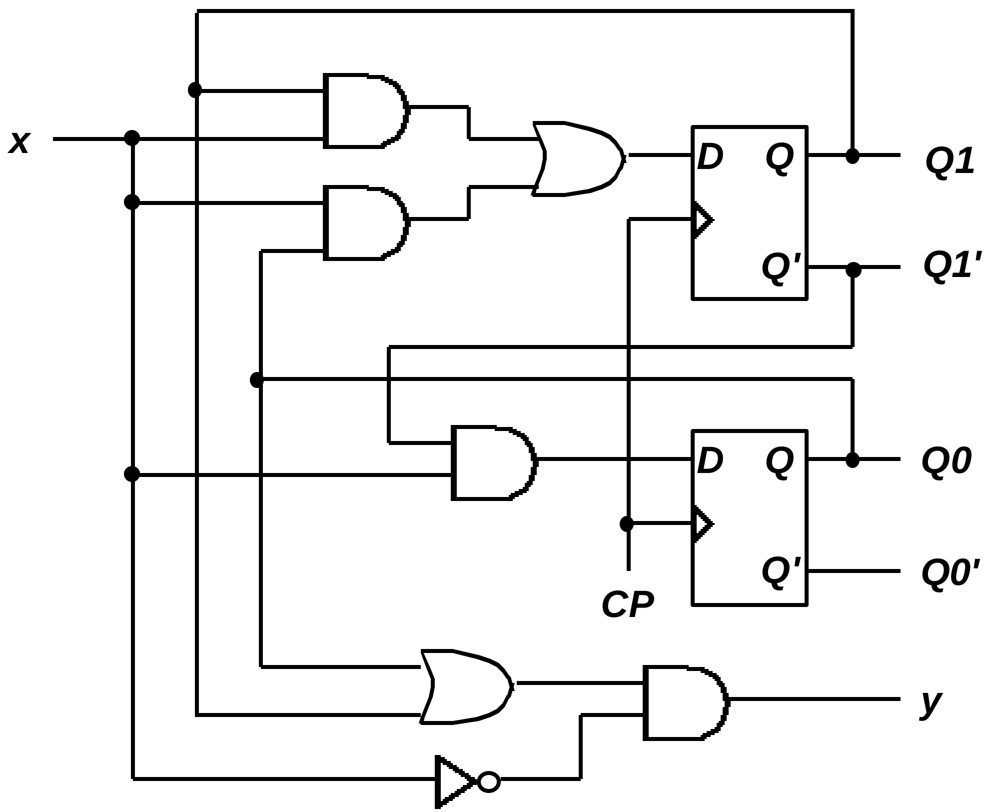
Mealy



Format:
Arc = input X / mealy output Y
Node = state

State Machine Analysis Example

Analyze the state machine:



1 Input (or excitation) equations:

D0 = Q1'. X

D1 = Q1 . x + Q0 . x

2 Characteristic equations:

Q0* = D0

Q1* = D1

Find State equations:

Q0* = Q1'. x

Q1* = Q1 . x + Q0 . x

3 Output equation:

y = (Q0 + Q1) . x'

This is a Mealy Machine since output = G(current state, input)

State Machine Analysis Example

4 **From the *state equations* and *output equation*, construct the *state transition/output table*:**

State equations:

Q0* = Q1'. x

Q1* = Q1 . x + Q0 . x

Output equation:

y = (Q0 + Q1) . x'

		x	
Q1	Q0	0	1
0	0	00,0	01,0
0	1	00,1	11,0
1	0	00,1	10,0
1	1	00,1	10,0

Q1* Q0* , y

Input (x)

Current State (Q1 Q0)

Next State when x =0

Output for current state when x =0

Next State when x =1

Output for current state when x =1

State Machine Analysis Example

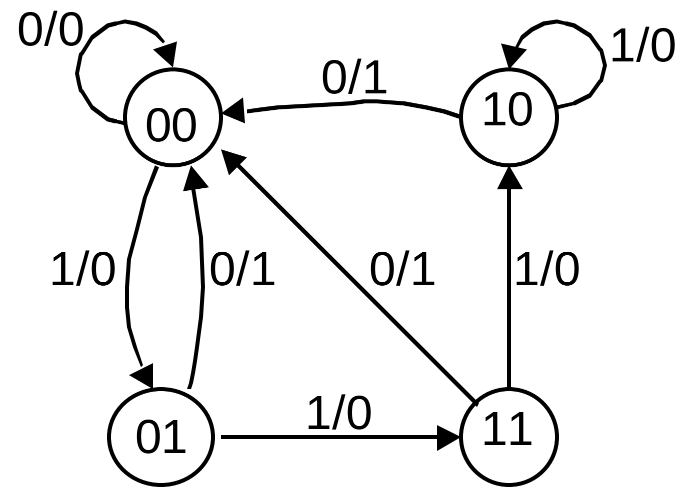
5 **Draw the state diagram of the state machine.**

state transition/output table

Q1	Q0	x = 0	x = 1
0	0	00,0	01,0
0	1	00,1	11,0
1	0	00,1	10,0
1	1	00,1	10,0

Q1* Q0* , y

state diagram



Arc = input x / output y
Node = state