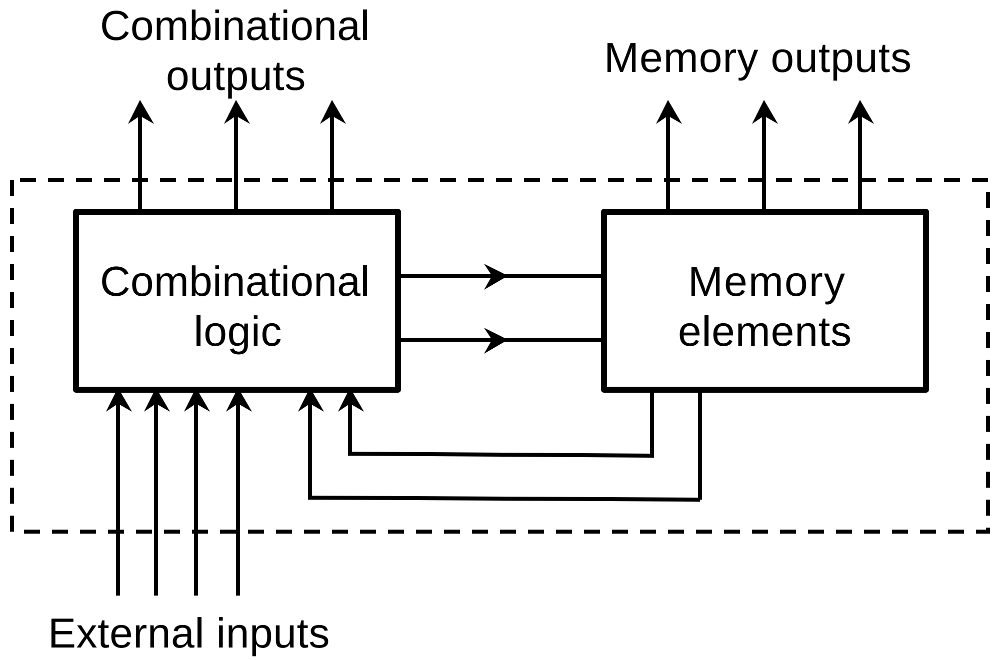
Types of Logic Circuits

- **Combinational logic circuits:**
  - Outputs depend only on its current inputs.
  - A combinational circuit may contain an arbitrary number of logic gates and inverters but no feedback loops.
    - A feedback loop is a connection from the output of one gate to propagate back into the input of that same gate
  - The function of a combinational circuit represented by a logic diagram is formally described using logic expressions and truth tables.
- **Sequential logic circuits:**
  - Outputs depend not only on the current inputs but also on the past sequences of inputs.
  - Sequential logic circuits contain combinational logic in addition to memory elements formed with feedback loops.
  - The behavior of sequential circuits is formally described with state transition tables and diagrams.

Sequential Circuits

- **The general structure of a sequential Circuit:**
  - **Combinational logic + Memory Elements**

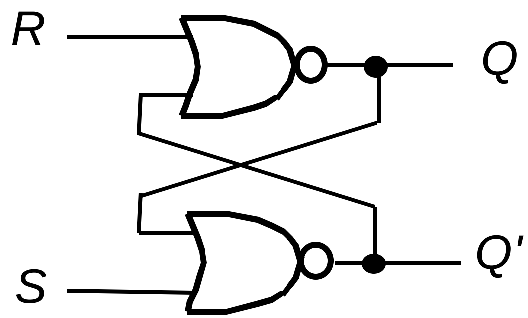


Memory element: a device that can remember value indefinitely, or change value on command from its inputs.

- **Examples: latches and flip-flops**

Memory Element Example: S-R (Set-Reset) Latch

- **The output Q represents the state of the latch**
- **When *Q* is HIGH, the latch is in *SET* state.**
- **When *Q* is LOW, the latch is in *RESET* state**



S-R latch using NOR gates

S	*R*	*Q*	*Q'*	
0	0	NC	NC	No change. Latch remained in present state.
1	0	1	0	Latch SET.
0	1	0	1	Latch RESET.
1	1	0	0	Invalid condition.

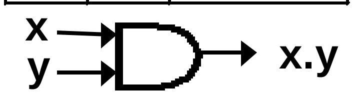
Characteristics or function table

- **Combinational Circuit Analysis:**
  - Start with a logic diagram of the circuit.
  - Proceed to a formal description of the function of the circuit using truth tables or logic expressions.
- **Combinational Circuit Synthesis:**
  - May start with an informal (possibly verbal) description of the function performed.
  - A formal description of the circuit function in terms of a truth table or logic expression.
  - The logic expression is manipulated using Boolean (or switching) algebra and optimized to minimize the number of gates needed, or to use specific type of gates.
  - A logic diagram is generated based on the resulting logic expression.

Boolean or Switching Algebra

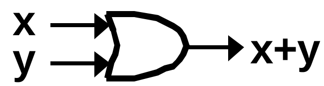
- **Set of Elements: {0,1}**
- **Set of Operations: { . , + , ' } AND (logical multiplication, .), OR (logical addition, +) , NOT**

x	y	x . y
0	0	0
0	1	0
1	0	0
1	1	1



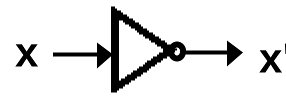
AND

x	y	x + y
0	0	0
0	1	1
1	0	1
1	1	1



OR

x	x'
0	1
1	0



NOT

- **Symbolic variables such as X used to represent the condition of a logic signal (0 or 1, low or high, on or off).**
- **Switching Algebra Axioms (or postulates):**
  - **Minimal set basic definitions (A1-A5, A1'-A5') that are assumed to be true and completely define switching algebra.**
  - **All other switching algebra theorems (T1-T15) can be proven using these axioms as a starting point.**

Switching Algebra Axioms

- **First two axioms state that a variable X can only take on only one of two values:**

(A1) $X = 0$ if $X \neq 1$

(A1') $X = 1$ if $X \neq 0$

- **Not Axioms, formally define X' (X prime or NOT X):**

(A2) If $X = 0$, then $X' = 1$

(A2') if $X = 1$, then, $X' = 0$

Note: Above axioms are stated in pairs with only difference being the interchange of the symbols 0 and 1.

Three More Switching Algebra Axioms

- The following three Boolean Algebra axioms state and formally define the AND, OR operations:

(A3)	$0 . 0 = 0$
(A3’)	$1 + 1 = 1$
(A4)	$1 . 1 = 1$
(A4’)	$0 + 0 = 0$
(A5)	$0 . 1 = 1 .0 = 0$
(A5’)	$1 + 0 = 0 + 1 = 1$

Axioms A1-A5, A1’-A5’ completely define switching algebra.

Switching Algebra: Single-Variables Theorems

- **Switching-algebra theorems are statements known to be always true (proven using axioms) that allow us to manipulate algebraic logic expressions to allow for simpler analysis.**

(e.g . X + 0 = X allow us to replace every X +0 with X)

The Theorems: (T1-T5, T1'-T5')

(T1)	X + 0 = X	(T1')	X . 1 = X	(Identities)
(T2)	X + 1 = 1	(T2')	X . 0 = 0	(Null elements)
(T3)	X + X = X	(T3')	X . X = X	(Idempotency)
(T4)	(X')' = X			(Involution)
(T5)	X + X' = 1	(T5')	X . X' = 0	(Complements)

Perfect Induction

- **Most theorems in switching algebra are simple to prove using *perfect induction*:**

  Since a switching variable can only take the values 0 and 1 we can prove a theorem involving a single variable X by proving it true for X = 0 and X =1

Example: To prove (T1) X + 0 = X

[X = 0] 0 + 0 = 0 true according to axiom A4'

[X = 1] 1 + 0 = 1 true according to axiom A5'

Switching Algebra:
Two- and Three-Variable Theorems

(Commutativity)

(T6) $X + Y = Y + X$

(T6') $X . Y = Y . X$

(Associativity)

(T7) $(X + Y) + Z = X + (Y + Z)$

(T7') $(X . Y) . Z = X . (Y . Z)$

T6-T7, T6' -T7' are similar to commutative and associative laws for addition and multiplication of integers and reals.

Two- and Three-Variable Theorems (Continued)

(Distributivity)

(T8) $X . Y + X . Z = X . (Y + Z)$

(T8') $(X + Y) . (X + Z) = X + Y . Z$

- **T8 allows to multiply-out an expression to get sum-of-products form** ***(distribute logical multiplication over logical addition):***

  For example:

  $V . (W + X) . (Y + Z) = V . W . Y + V . W . Z + V . X . Y + V . X . Z$

  sum-of-products form

- **T8' allows to add-out an expression to get a product-of-sums form** ***(distribute logical addition over logical multiplication)*****:**

  For example:

$(V . W . X) + (Y . Z) = (V + Y) . (V + Z) . (W + Y) . (W + Z) . (X + Y) . (X + Z)$

product-of-sums form

Theorem Proof using Truth Table

- **Can use truth table to prove T8 by perfect induction.**
- **i.e Prove that: X . Y + X . Z = X . (Y + Z)**

(i) Construct truth table for both sides of above equality.

x	y	z	y + z	x.(y + z)	x.y	x.z	x.y + x.z
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

(ii) Check that from truth table check that that X . Y + X . Z = X . (Y + Z)

This is satisfied because output column values for X . Y + X . Z and output column values for X . (Y + Z) are equal for all cases.

Two- and Three-Variable Theorems (Continued)

(Covering)

$$\text{(T9)} \quad X + X \cdot Y = X$$

$$\text{(T9')} \quad X \cdot (X + Y) = X$$

(Combining)

$$\text{(T10)} \quad X \cdot Y + X \cdot Y' = X$$

$$\text{(T10')} \quad (X + Y) \cdot (X + Y') = X$$

- **T9-T10 used in the minimization of logic functions.**

Two- and Three-Variable Theorems (Continued)

(Consensus)

(T11) $X . Y + X'. Z + Y . Z = X . Y + X' . Z$

(T11') $(X + Y) . (X' + Z) . (Y + Z) = (X + Y) . (X' + Z)$

- **In T11 the term Y. Z is called the consensus of the term X . Y and the term X' . Z:**
  - **If Y . Z = 1, then either X . Y or X' . Z must also be 1.**
  - **Thus the term Y . Z is redundant and may be dropped.**

n-Variable Theorems

(Generalized idempotency)

(T12) $X + X + \ ...\ + X = X$

(T12') $X \cdot X \cdot \ ...\ \cdot X = X$

(DeMorgan's theorems)

(T13) $(X_1 \cdot X_2 \cdot \ ....\ \cdot X_n)' = X_1' + X_2' + ... + X_n'$

(T13') $(X_1 + X_2 + \ ...\ + X_n)' = X_1' \cdot X_2' \cdot .... \cdot X_n'$

(T13), (T13') are probably the most commonly used theorems of switching algebra.

Examples Using DeMorgan's theorems

Example: Equivalence of NAND Gate:

A two-input NAND Gate has the output expression

Z = (X . Y)' using (T13)   Z = (X . Y)' = (X' + Y')

The function of a NAND gate can be achieved with an OR gate with an inverter at each input.

Example: Equivalence of NOR Gate

A two-input NOR Gate has the output expression Z=(X+Y)'

using (T13')   Z = (X + Y)' = X' . Y'

The function of a NOR gate can be achieved with an AND gate with an inverter at each input.

n-Variable Theorems (Continued)

(Generalized DeMorgran's theorem)

(T14) $[F(X_1, X_2, . . ., X_n, +, .)]' = F(X_1', X_2', . . ., X_n', . , +)$

- **States that given any n-variable logic expression its complement can be found by swapping + and . and complementing all variables.**

Example:

$F(W,X,Y,Z) = (W'.X) + (X.Y) + (W.(X' + Z'))$

$= ((W)' . X) + (X. Y) + (W.((X)' + (Z)'))$

$[F(W,X,Y,Z)]' = ((W')' + X') .(X' + Y').(W' + ((X')'.(Z')'))$

Using T4, $(X')' = X$ **simplifies it to:**

$[F(W,X,Y,Z)]' = (W + X') . (X' + Y') . (W' + (X . Z))$