Clocked Synchronous State-machine Analysis

Given the circuit diagram of a state machine:

1 **Analyze the combinational logic to determine flip-flop input (excitation) equations:** $D_i = F_i$ **(Q, inputs)**
   - **The input to each flip-flop is based upon current state and circuit inputs.**

2 **Substitute excitation equations into flip-flop characteristic equations, giving transition equations:** $Q_i^* = H_i(D_i)$

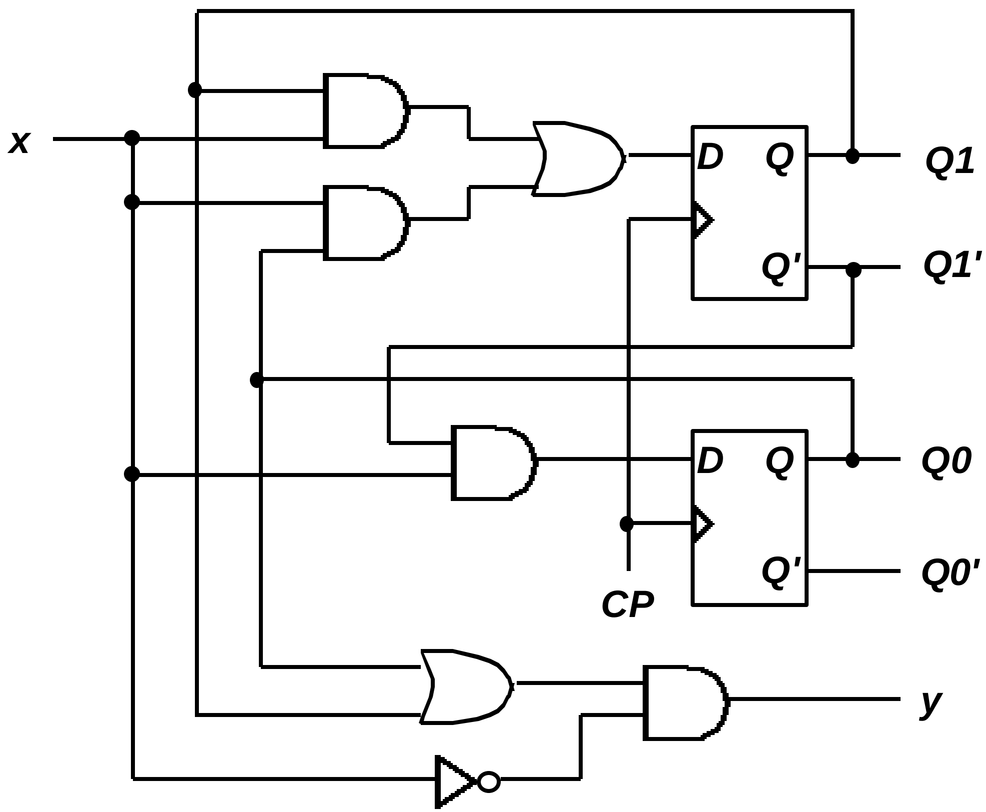
3 **From the circuit, find output equations:** **Z = G (Q, inputs)**
   - **The outputs are based upon the current state and possibly the inputs.**

4 **Construct a state transition/output table from the transition and output equations:**
   - **Similar to truth table.**
   - **Present state on the left side.**
   - **Outputs and next state for each input value on the right side.**
   - **Provide meaningful names for the states in state table, if possible.**

5 **Draw the state diagram which is the graphical representation of state table.**

State Machine Analysis Example

Analyze the state machine:



1 Input (or excitation) equations:

D0 = Q1'. X

D1 = Q1 . x + Q0 . x

2 Characteristic equations:

Q0* = D0

Q1* = D1

Find State equations:

Q0* = Q1'. x

Q1* = Q1 . x + Q0 . x

3 Output equation:

y = (Q0 + Q1) . x'

This is a Mealy Machine since output = G(current state, input)

State Machine Analysis Example

4 **From the *state equations* and *output equation*, construct the *state transition/output table*:**

State equations:

$$Q0^* = Q1' . x$$

$$Q1^* = Q1 . x + Q0 . x$$

Output equation:

$$y = (Q0 + Q1) . x'$$

		x = 0	x = 1
Q1	Q0	0	1
0	0	00,0	01,0
0	1	00,1	11,0
1	0	00,1	10,0
1	1	00,1	10,0

Q1* Q0* , y

Input (x)

Current State (Q1 Q0)

Next State when x =0

Output for current state when x =0

Next State when x =1

Output for current state when x =1

State Machine Analysis Example

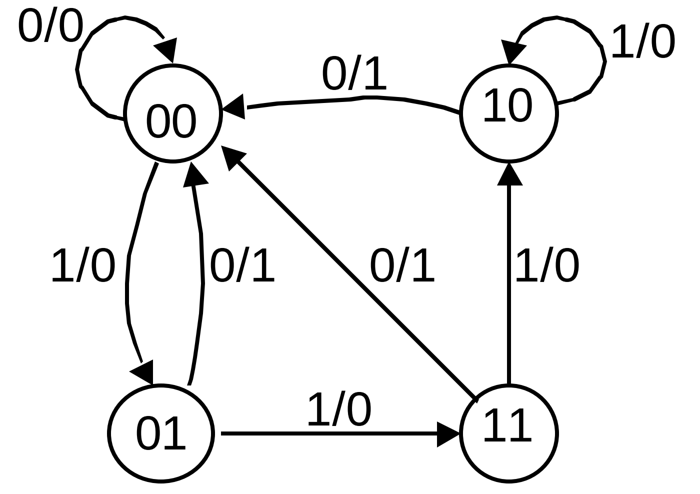
5 **Draw the state diagram of the state machine.**

state transition/output table

Q1	Q0	x = 0	x = 1
0	0	00,0	01,0
0	1	00,1	11,0
1	0	00,1	10,0
1	1	00,1	10,0

Q1* Q0* , y

state diagram



Arc = input x / output y
Node = state

Clocked State-machine Analysis: State Naming

- **State Naming:**
  - Optionally name the states and substitute state names S for state-variable combinations in transition/output table and in state diagram.
  - Example: For a circuit with two flip-flops:

Q1 Q0	State Name
0 0	A
0 1	B
1 0	C
1 1	D

Clocked State-machine Analysis Example: Transition/Output Table Using State Names

For the last example naming The States:

Q1	Q0	State Name
0	0	A
0	1	B
1	0	C
1	1	D

Transition/output Table:

			x = 0	x = 1
	Q1	Q0	0	1
A	0	0	00,0	01,0
B	0	1	00,1	11,0
C	1	0	00,1	10,0
D	1	1	00,1	10,0

Q1* Q0* , y

Transition/output Table using state names:

S	x = 0	x = 1
A	A,0	B,0
B	A,1	D,0
C	A,1	C,0
D	A,1	C,0

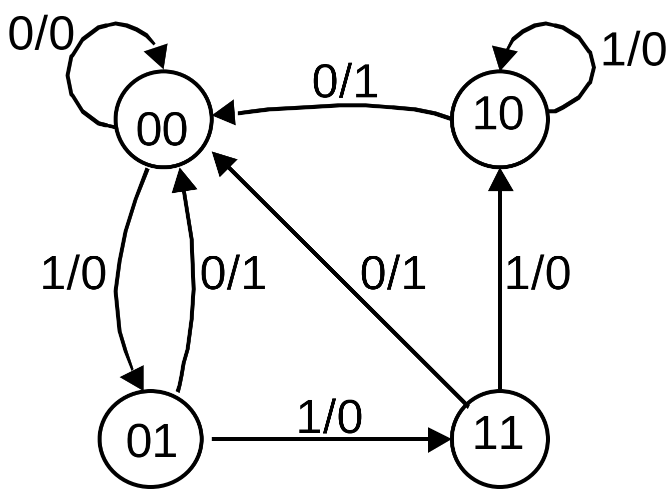
S* , y

Clocked State-machine Analysis Example: State Diagram Using State Naming

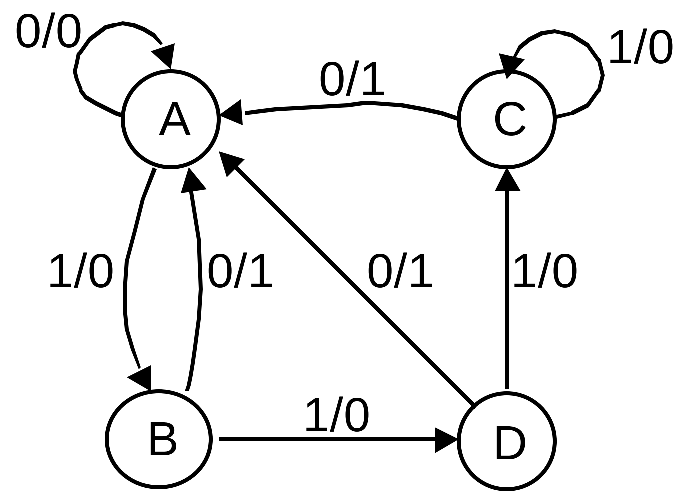
Naming The States:

Q1	Q0	State Name
0	0	A
0	1	B
1	0	C
1	1	D

State Diagram without state naming:



State Diagram with state naming:

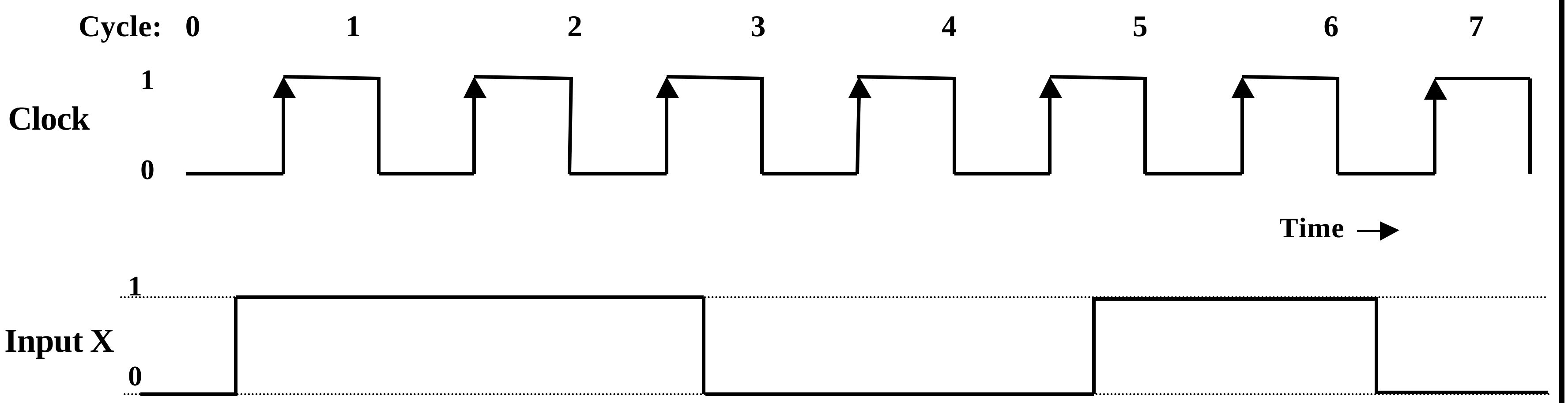


Arc = input x / output y
Node = state

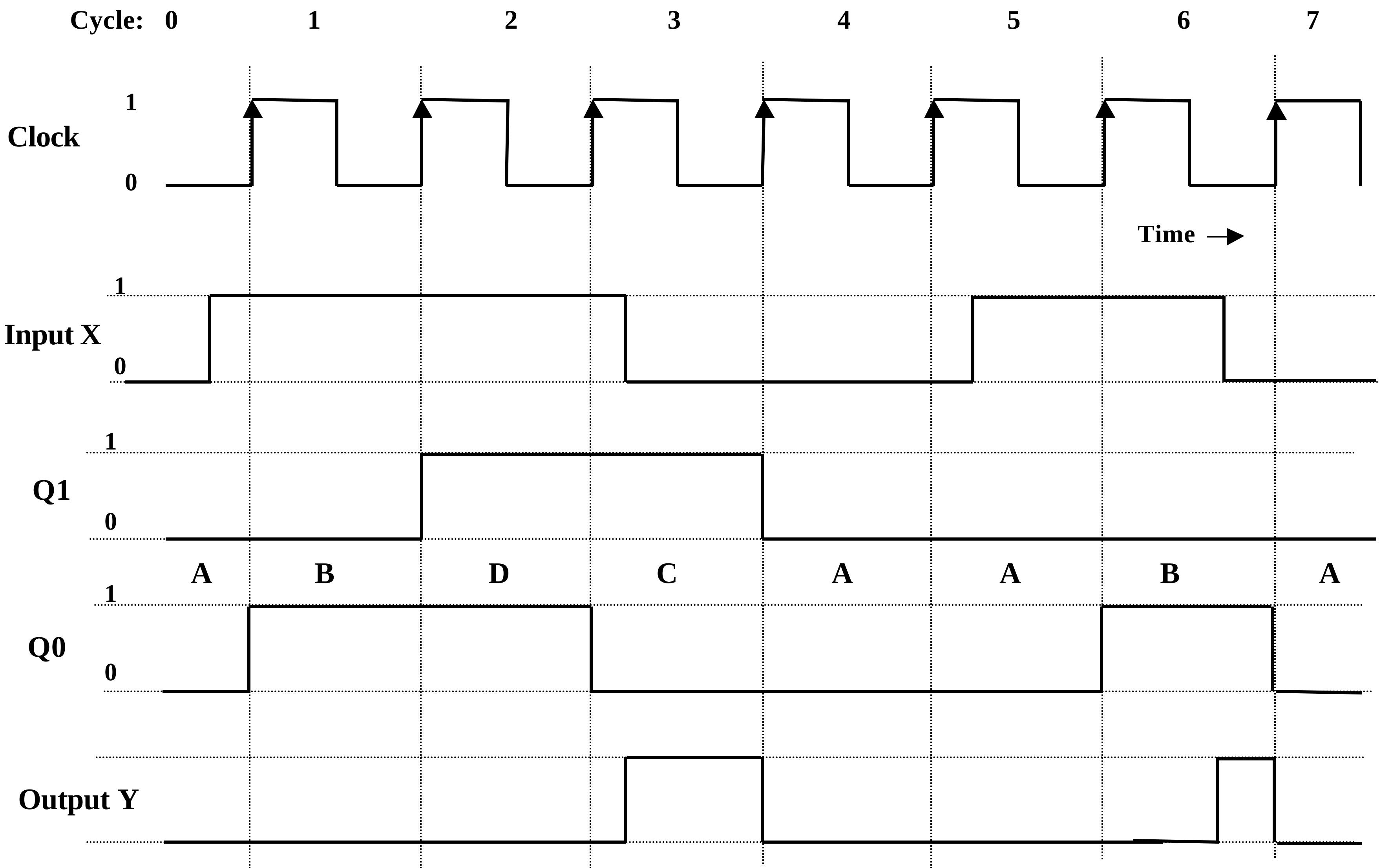
Clocked State-machine Analysis:
State Machine Timing Diagram

- **The timing diagram for a state machine graphically shows the state machine response in terms of state variables and output signals vs. time for given time-varying input signals and a given initial state.**
- **State machine timing diagrams can be generated using transition/output tables or state diagrams.**
- **Timing diagrams can be used to account for both combinational and flip-flop propagation delays.**

- **Example: For the state machine in the previous example show the timing diagram for the following input, assuming an initial state A and ignoring propagation delays:**

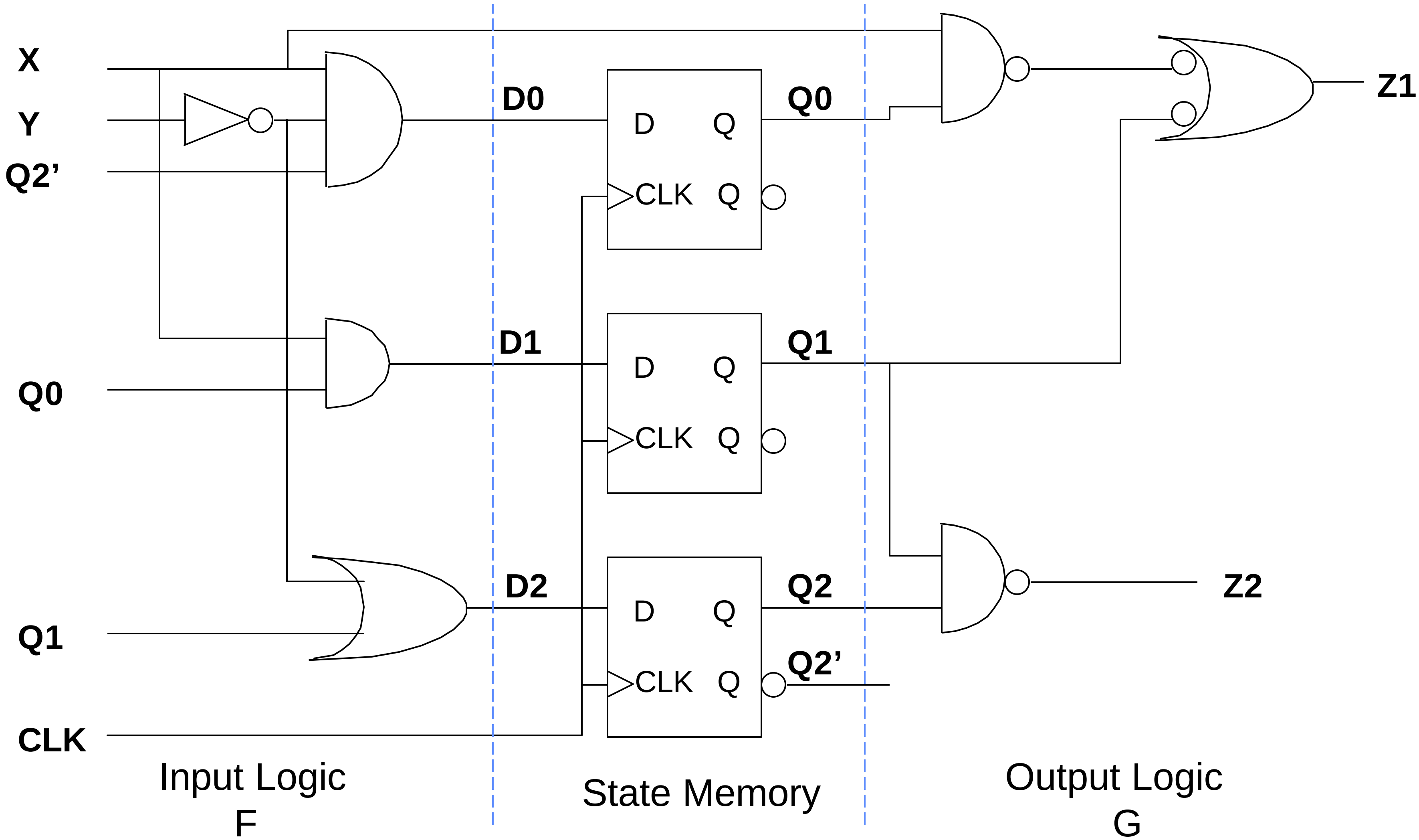


State Machine Timing Diagram Example



State Machine Analysis Example 2

Analyze the state machine:



State Machine Analysis Example 2

1

Excitation Equations

$D0 = X . Y'.Q2$

$D1 = X . Q0$

$D2 = Y' + Q1$

2

Characteristic Equations

$Q0^* = D0$

$Q1^* = D1$

$Q2^* = D2$

State or Transition Equations

$Q0^* = D0 = X . Y' . Q2'$

$Q1^* = D1 = X . Q0$

$Q2^* = D2 = Y' + Q1$

3

Output Equations

$Z1 = X . Q0 + Q1'$

$Z2 = (Q1 . Q2)'$

State Machine Analysis Example 2

4 **From the *state equations* and *output equation*, construct the *state transition/output table*:**

state name	Q2 Q1 Q0	XY 00	01	11	10
A	0 0 0	100, 11	000, 11	000, 11	101, 11
B	0 0 1	100, 11	000, 11	010, 11	111, 11
C	0 1 0	100, 01	100, 01	100, 01	101, 01
D	0 1 1	100, 01	100, 01	110, 11	111, 11
E	1 0 0	100, 11	000, 11	000, 11	100, 11
F	1 0 1	100, 11	000, 11	010, 11	110, 11
G	1 1 0	100, 00	100, 00	100, 00	100, 00
H	1 1 1	100, 00	100, 00	110, 10	110,10

Q2* Q1* Q0*, Z1 Z2
(Next State, Outputs)

Transition Equations

Q0* = D0 = X . Y' . Q2'

Q1* = D1 = X . Q0

Q2* = D2 = Y' + Q1

Output Equations

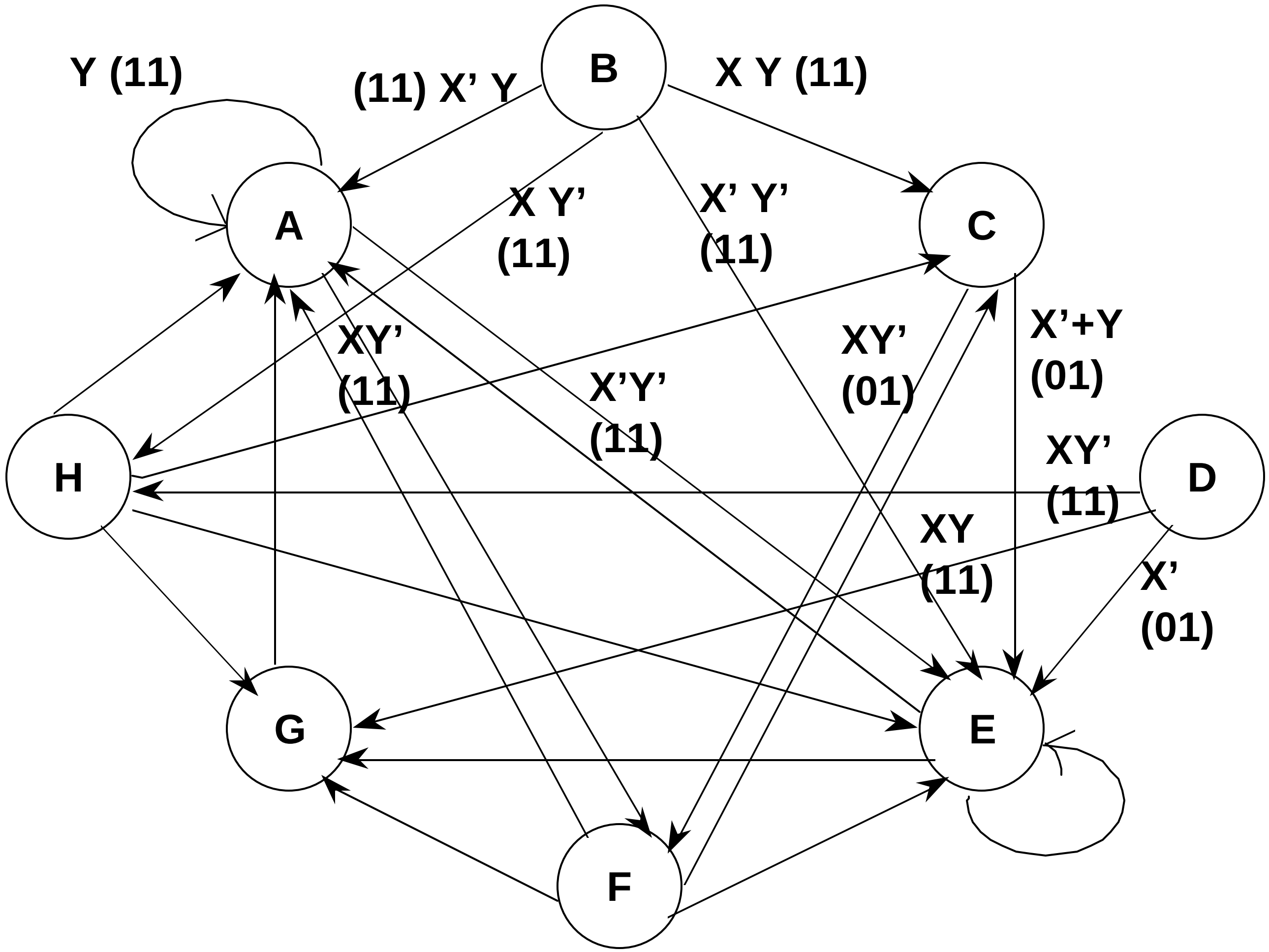
Z1 = X . Q0 + Q1'

Z2= (Q1 . Q2)'

State-machine Analysis Example 2: Transition/Output Table Using State Names

S	XY = 00	XY = 01	XY = 11	XY = 10
A	E, 11	A, 11	A, 11	F, 11
B	E, 11	A, 11	C, 11	H, 11
C	E, 01	E, 01	E, 01	F, 01
D	E, 01	E, 01	G, 11	H, 11
E	E, 11	A, 11	A, 11	E, 11
F	E, 11	A, 11	C, 11	G, 11
G	E, 00	E, 00	E, 00	E, 00
H	E, 00	E, 00	G, 10	G,10
	S*, Z1 Z2			

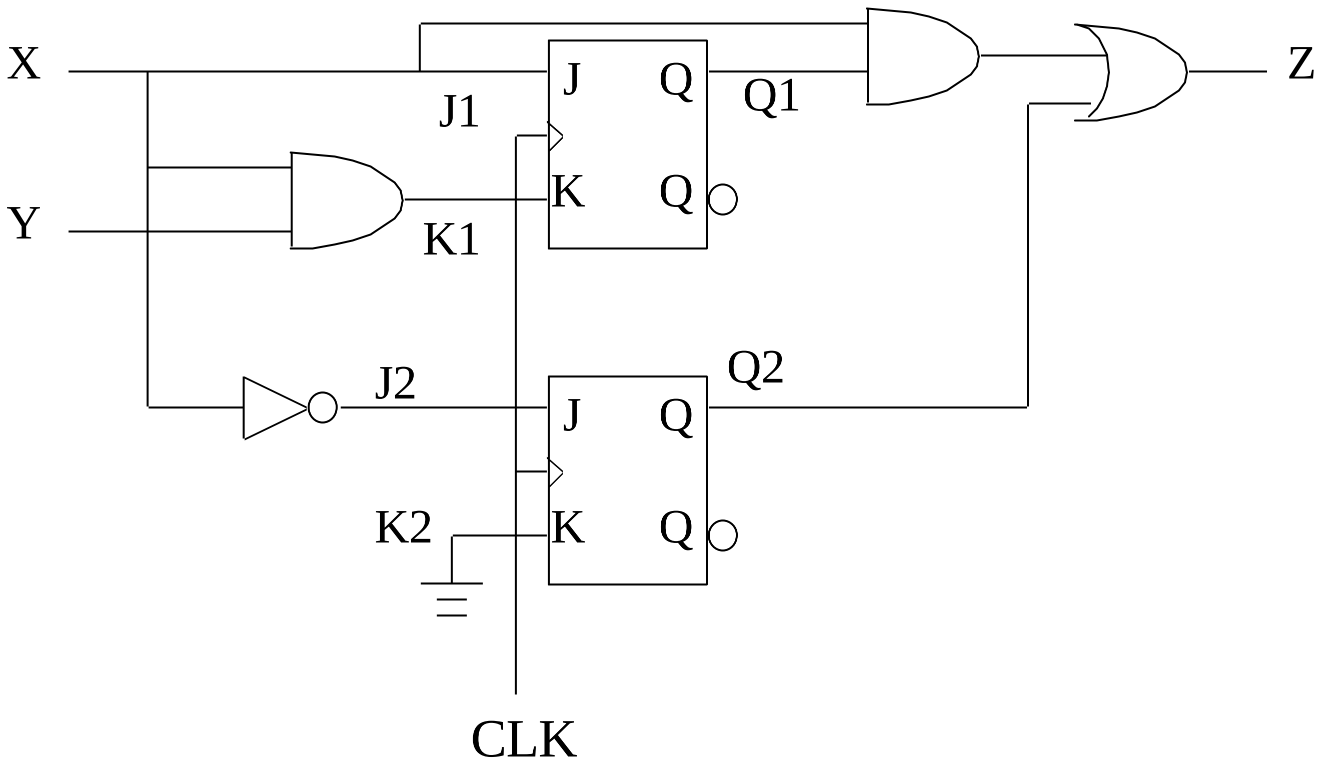
State-machine Analysis Example 2: State Diagram (incomplete)



Arc: input expression (outputs) = expression (Z1 /Z2)

State Machine Analysis Example 3

Analyze the state machine:



State Machine Analysis Example 3

Excitation Equations

$J1 = X$

$K1 = X \cdot Y$

$J2 = X'$

$K2 = 0$

1

Characteristic Equations

$Q^* = J \cdot Q' + K' \cdot Q$

$Q1^* = J1 \cdot Q1' + K1' \cdot Q1$

$Q2^* = J2 \cdot Q2' + K2' \cdot Q2$

2

Transition Equations

$Q1^* = X \cdot Q1' + (X \cdot Y)' \cdot Q1 = X \cdot Q1' + X' \cdot Q1 + Y' \cdot Q1$

$Q2^* = X' \cdot Q2' + 0' \cdot Q2 = X' \cdot Q2' + Q2$

3

Output Equation

$Z = X \cdot Q1 + Q2$

State Machine Analysis Example 3

4 **From the *state equations* and *output equation*, construct the *state transition/output table*:**

S	Q1 Q2	XY = 00	01	11	10
A	0 0	01,0	01,0	10,0	10,0
B	0 1	01,1	01,1	11,1	11,1
C	1 0	11,0	11,0	00,1	10,1
D	1 1	11,1	11,1	01,1	11,1
		Q1* Q2*, Z			

Transition Equations

Q1* = X·Q1' + X'·Q1 + Y'·Q1

Q2* = X'·Q2' + Q2

Output Equation

Z = X·Q1 + Q2

State-machine Analysis Example 3: Transition/Output Table Using State Names

S	XY 00	01	11	10
A	B,0	B,0	C,0	C,0
B	B,1	B,1	D,1	D,1
C	D,0	D,0	A,1	C,1
D	D,1	D,1	B,1	D,1

S*, Z

State-machine Analysis Example 3: State Diagram

Arc Format:
inputs xy
output z