Parallelization of An Example Program

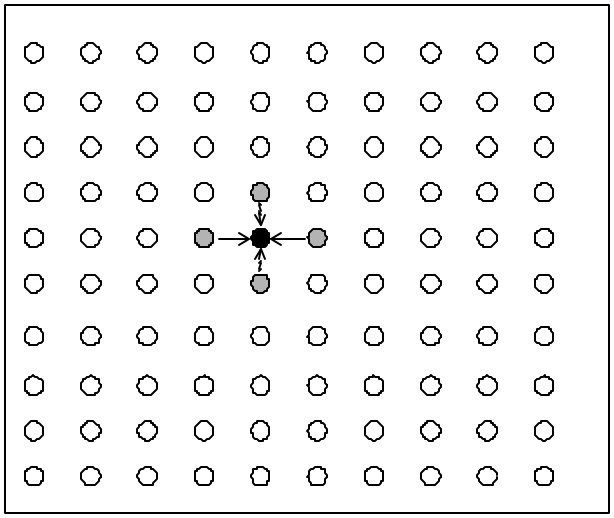
Examine a simplified version of a piece of Ocean simulation

- Iterative equation solver

Illustrate parallel program in low-level parallel language

- C-like pseudocode with simple extensions for parallelism
- Expose basic communication and synch. primitives that must be supported

Grid Solver Example



Expression for updating each interior point:

$$A[i,j] = 0.2 \times (A[i,j] + A[i,j-1] + A[i-1,j] + A[i,j+1] + A[i+1,j])$$

- Simplified version of solver in Ocean simulation
- Gauss-Seidel (near-neighbor) sweeps to convergence
  - interior n-by-n points of (n+2)-by-(n+2) updated in each sweep
  - updates done in-place in grid, and diff. from prev. value computed
  - accumulate partial diffs into global diff at end of every sweep
  - check if error has converged (to within a tolerance parameter)
  - if so, exit solver; if not, do another sweep

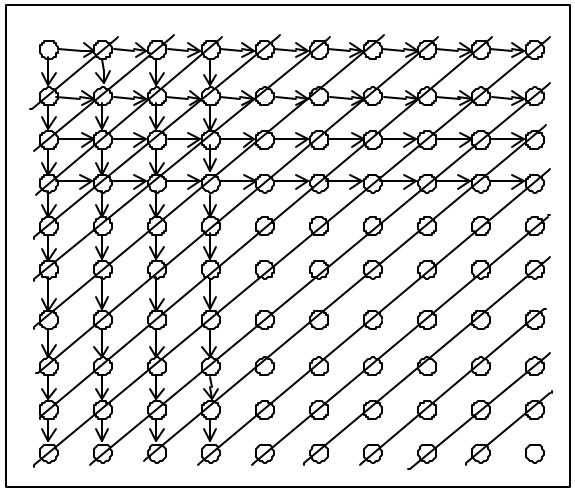
```
1.  int n;                                  /*size of matrix: (n + 2-by-n + 2) elements*/
2.  float **A, diff = 0;

3.  main()
4.  begin
5.     read(n) ;                            /*read input parameter: matrix size*/
6.     A ← malloc (a 2-d array of size n + 2 by n + 2 doubles);
7.     initialize(A);                       /*initialize the matrix A somehow*/
8.     Solve (A);                           /*call the routine to solve equation*/
9.  end main

10. procedure Solve (A)                     /*solve the equation system*/
11.    float **A;                           /*A is an (n + 2)-by-(n + 2) array*/
12. begin
13.    int i, j, done = 0;
14.    float diff = 0, temp;
15.    while (!done) do                     /*outermost loop over sweeps*/
16.       diff = 0;                         /*initialize maximum difference to 0*/
17.       for i ← 1 to n do                 /*sweep over nonborder points of grid*/
18.          for j ← 1 to n do
19.             temp = A[i,j];             /*save old value of element*/
20.             A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                A[i,j+1] + A[i+1,j]);   /*compute average*/
22.             diff += abs(A[i,j] - temp);
23.          end for
24.       end for
25.       if (diff/(n*n) < TOL) then done = 1;
26.    end while
27. end procedure
```

Decomposition

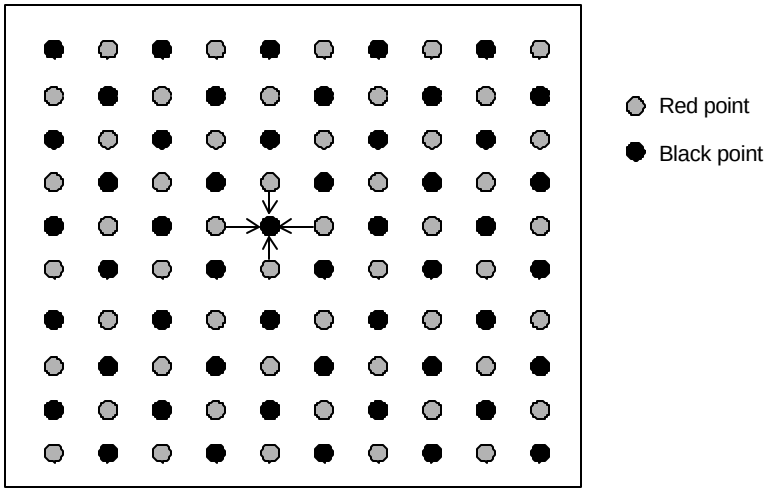
- Simple way to identify concurrency is to look at loop iterations
  - *dependence analysis*; if not enough concurrency, then look further
- Not much concurrency here at this level (all loops *sequential*)
- Examine fundamental dependences, ignoring loop structure



- Concurrency *O(n)* along anti-diagonals, serialization *O(n)* along diag.
- Retain loop structure, use pt-to-pt synch; Problem: too many synch ops.
- Restructure loops, use global synch; imbalance and too much synch

Exploit Application Knowledge

- Reorder grid traversal: red-black ordering



- Different ordering of updates: may converge quicker or slower
- Red sweep and black sweep are each fully parallel:
- Global synch between them (conservative but convenient)
- Ocean uses red-black; we use simpler, asynchronous one to illustrate
  - no red-black, simply ignore dependences within sweep
  - sequential order same as original, parallel program *nondeterministic*

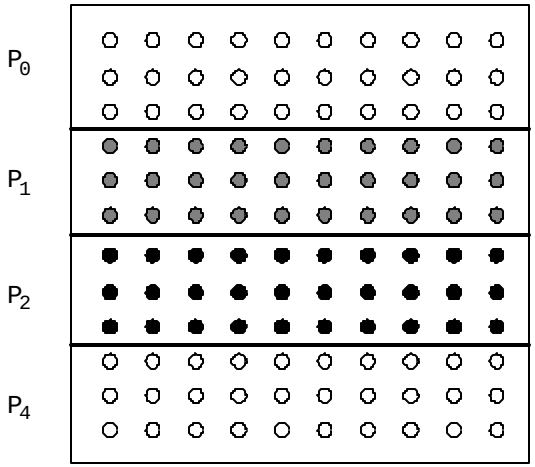
Decomposition Only

```
15. while (!done) do                          /*a sequential loop*/
16.   diff = 0;
17.   for_all i ← 1 to n do                  /*a parallel loop nest*/
18.     for_all j ← 1 to n do
19.       temp = A[i,j];
20.       A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.         A[i,j+1] + A[i+1,j]);
22.       diff += abs(A[i,j] - temp);
23.     end for_all
24.   end for_all
25.   if (diff/(n*n) < TOL) then done = 1;
26. end while
```

- Decomposition into elements: degree of concurrency n^2
- To decompose into rows, make line 18 loop sequential; degree n
- `for_all` leaves assignment left to system
  - but implicit global synch. at end of `for_all` loop

Assignment

- Static assignments (given decomposition into rows)
  - block assignment of rows: Row i is assigned to process $\left\lfloor \frac{i}{p} \right\rfloor$
  - cyclic assignment of rows: process i is assigned rows i, $i+p$, and so on



- Dynamic assignment
  - get a row index, work on the row, get a new row, and so on
- Static assignment into rows reduces concurrency (from n to p)
  - block assign. reduces communication by keeping adjacent rows together
- Let's dig into orchestration under three programming models

Data Parallel Solver

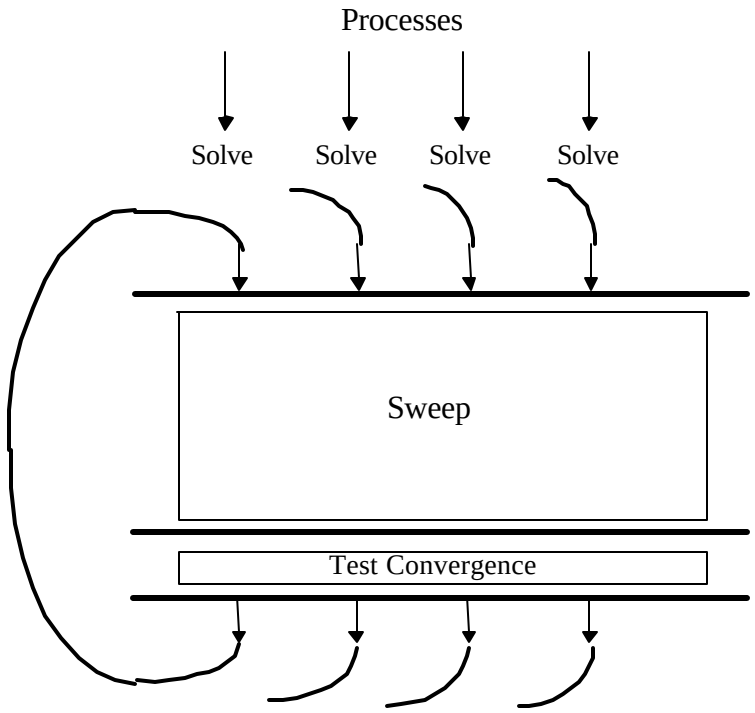
```
1.  int n, nprocs;                          /*grid size (n + 2-by-n + 2) and number of processes*/
2.  float **A, diff = 0;

3.  main()
4.  begin
5.    read(n); read(nprocs);      ;         /*read input grid size and number of processes*/
6.    A ← G_MALLOC (a 2-d array of size n+2 by n+2 doubles);
7.    initialize(A);                        /*initialize the matrix A somehow*/
8.    Solve (A);                            /*call the routine to solve equation*/
9.  end main

10. procedure Solve(A)                      /*solve the equation system*/
11.     float **A;                          /*A is an (n + 2-by-n + 2) array*/
12.   begin
13.   int i, j, done = 0;
14.   float mydiff = 0, temp;
14a.    DECOMP A[BLOCK,*, nprocs];
15.   while (!done) do                      /*outermost loop over sweeps*/
16.     mydiff = 0;                         /*initialize maximum difference to 0*/
17.     for_all i ← 1 to n do               /*sweep over non-border points of grid*/
18.       for_all j ← 1 to n do
19.         temp = A[i,j];                  /*save old value of element*/
20.         A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.           A[i,j+1] + A[i+1,j]);         /*compute average*/
22.         mydiff += abs(A[i,j] - temp);
23.       end for_all
24.     end for_all
24a.      REDUCE (mydiff, diff, ADD);
25.     if (diff/(n*n) < TOL) then done = 1;
26.   end while
27. end procedure
```

Shared Address Space Solver

Single Program Multiple Data (SPMD)



- Assignment controlled by values of variables used as loop bounds

```
1.       int n, nprocs;            /*matrix dimension and number of processors to be used*/
2a.      float **A, diff;          /*A is global (shared) array representing the grid*/
                                   /*diff is global (shared) maximum difference in current
                                   sweep*/
2b.      LOCKDEC(diff_lock);       /*declaration of lock to enforce mutual exclusion*/
2c.      BARDEC (bar1);            /*barrier declaration for global synchronization between
                                   sweeps*/

3.    main()
4.    begin
5.          read(n); read(nprocs);      /*read input matrix size and number of processes*/
6.          A ← G_MALLOC (a two-dimensional array of size n+2 by n+2 doubles);
7.          initialize(A);              /*initialize A in an unspecified way*/
8a.         CREATE (nprocs-1, Solve, A);
8.       Solve(A);                      /*main process becomes a worker too*/
8b.         WAIT_FOR_END (nprocs-1);    /*wait for all child processes created to terminate*/
9.    end main

10.   procedure Solve(A)
11.      float **A;                                  /*A is entire n+2-by-n+2 shared array,
                                                     as in the sequential program*/
12.   begin
13.      int i,j, pid, done = 0;
14.      float temp, mydiff = 0;                     /*private variables*/
14a.     int mymin = 1 + (pid * n/nprocs);           /*assume that n is exactly divisible by*/
14b.     int mymax = mymin + n/nprocs - 1            /*nprocs for simplicity here*/

15.      while (!done) do                  /*outer loop over all diagonal elements*/
16.         mydiff = diff = 0;             /*set global diff to 0 (okay for all to do it)*/
16a.     BARRIER(bar1, nprocs);            /*ensure all reach here before anyone modifies diff*/
17.         for i ← mymin to mymax do      /*for each of my rows*/
18.               for j ← 1 to n do        /*for all nonborder elements in that row*/
19.               temp = A[i,j];
20.               A[i,j] = 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                  A[i,j+1] + A[i+1,j]);
22.               mydiff += abs(A[i,j] - temp);
23.            endfor
24.         endfor
25a.        LOCK(diff_lock);               /*update global diff if necessary*/
25b.        diff += mydiff;
25c.        UNLOCK(diff_lock);
25d.        BARRIER(bar1, nprocs);         /*ensure all reach here before checking if done*/
25e.        if (diff/(n*n) < TOL) then done = 1;     /*check convergence; all get
                                                     same answer*/
25f.        BARRIER(bar1, nprocs);
26.      endwhile
27.   end procedure
```

Notes on SAS Program

- SPMD: not lockstep or even necessarily same instructions

- Assignment controlled by values of variables used as loop bounds
  - unique pid per process, used to control assignment

- Done condition evaluated redundantly by all

- Code that does the update identical to sequential program
  - each process has private mydiff variable

- Most interesting special operations are for synchronization
  - accumulations into shared diff have to be mutually exclusive
  - why the need for all the barriers?

Need for Mutual Exclusion

- Code each process executes:

```
load the value of diff into register r1
add the register r2 to register r1
store the value of register r1 into diff
```

- A possible interleaving:

P1	P2	
		{P1 gets 0 in its r1}
	r1 ← diff	{P2 also gets 0}
r1 ← r1+r2		{P1 sets its r1 to 1}
	r1 ← r1+r2	{P2 sets its r1 to 1}
diff ← r1		{P1 sets cell_cost to 1}
	diff ← r1	{P2 also sets cell_cost to 1}

- Need the sets of operations to be atomic (mutually exclusive)

Mutual Exclusion

Provided by LOCK-UNLOCK around *critical section*

- Set of operations we want to execute atomically
- Implementation of LOCK/UNLOCK must guarantee mutual excl.

Can lead to significant serialization if contended

- Especially since expect non-local accesses in critical section
- Another reason to use private mydiff for partial accumulation

Global Event Synchronization

BARRIER(nprocs): wait here till nprocs processes get here

- Built using lower level primitives
- Global sum example: wait for all to accumulate before using sum
- Often used to separate phases of computation

Process P_1	*Process P_2*	*Process P_nprocs*
set up eqn system	set up eqn system	set up eqn system
Barrier (name, nprocs)	**Barrier** (name, nprocs)	**Barrier** (name, nprocs)
solve eqn system	solve eqn system	solve eqn system
Barrier (name, nprocs)	**Barrier** (name, nprocs)	**Barrier** (name, nprocs)
apply results	apply results	apply results
Barrier (name, nprocs)	**Barrier** (name, nprocs)	**Barrier** (name, nprocs)

- Conservative form of preserving dependences, but easy to use

WAIT_FOR_END (nprocs-1)

Pointt-to-point Event Synch (Not Used Here)

One process notifies another of an event so it can proceed

- Common example: producer-consumer (bounded buffer)
- Concurrent programming on uniprocessor: semaphores
- Shared address space parallel programs: semaphores, or use ordinary variables as flags

P_1	P_2
	`A = 1;`
	`b: flag = 1;`
`a: while (flag is 0) do nothing;`	
`print A;`	

- *Busy-waiting* or *spinning*

Group Event Synchronization

Subset of processes involved

- Can use flags or barriers (involving only the subset)
- Concept of producers and consumers

Major types:

- Single-producer, multiple-consumer
- Multiple-producer, single-consumer
- Multiple-producer, single-consumer

Message Passing Grid Solver

- Cannot declare A to be shared array any more
- Need to compose it logically from per-process private arrays
  - usually allocated in accordance with the assignment of work
  - process assigned a set of rows allocates them locally
- Transfers of entire rows between traversals
- Structurally similar to SAS (e.g. SPMD), but orchestration different
  - data structures and data access/naming
  - communication
  - synchronization

```
1. int pid, n, b;                          /*process id, matrix dimension and number of
                                             processors to be used*/
2. float **myA;
3. main()
4. begin
5.     read(n);   read(nprocs);            /*read input matrix size and number of processes*/
8a.    CREATE (nprocs-1, Solve);
8b.    Solve();                            /*main process becomes a worker too*/
8c.    WAIT_FOR_END (nprocs-1);            /*wait for all child processes created to terminate*/
9. end main

10.  procedure Solve()
11.  begin
13.    int i,j, pid, n' = n/nprocs, done = 0;
14.    float temp, tempdiff, mydiff = 0;   /*private variables*/
6.   myA ← malloc(a 2-d array of size [n/nprocs + 2] by n+2);
                                           /*my assigned rows of A*/
7. initialize(myA);                        /*initialize my rows of A, in an unspecified way*/

15.  while (!done) do
16.    mydiff = 0;                         /*set local diff to 0*/
16a.   if (pid != 0) then SEND(&myA[1,0],n*sizeof(float),pid-1,ROW);
16b.   if (pid = nprocs-1) then
         SEND(&myA[n',0],n*sizeof(float),pid+1,ROW);
16c.   if (pid != 0) then RECEIVE(&myA[0,0],n*sizeof(float),pid-1,ROW);
16d.   if (pid != nprocs-1) then
         RECEIVE(&myA[n'+1,0],n*sizeof(float), pid+1,ROW);
                                           /*border rows of neighbors have now been copied
                                             into myA[0,*] and myA[n'+1,*]*/
17.    for i ← 1 to n' do                  /*for each of my (nonghost) rows*/
18.      for j ← 1 to n do                 /*for all nonborder elements in that row*/
19.        temp = myA[i,j];
20.        myA[i,j] = 0.2 * (myA[i,j] + myA[i,j-1] + myA[i-1,j] +
21.          myA[i,j+1] + myA[i+1,j]);
22.        mydiff += abs(myA[i,j] - temp);
23.      endfor
24.    endfor
                                           /*communicate local diff values and determine if
                                             done; can be replaced by reduction and broadcast*/
25a.     if (pid != 0) then                /*process 0 holds global total diff*/
25b.       SEND(mydiff,sizeof(float),0,DIFF);
25c.       RECEIVE(done,sizeof(int),0,DONE);
25d.     else                              /*pid 0 does this*/
25e.       for i ← 1 to nprocs-1 do        /*for each other process*/
25f.         RECEIVE(tempdiff,sizeof(float),*,DIFF);
25g.       mydiff += tempdiff;             /*accumulate into total*/
25h.     endfor
25i      if (mydiff/(n*n) < TOL) then      done = 1;
25j.       for i ← 1 to nprocs-1 do        /*for each other process*/
25k.         SEND(done,sizeof(int),i,DONE);
25l.       endfor
25m.   endif
26.  endwhile
27.  end procedure
```

Notes on Message Passing Program

- Use of ghost rows
- Receive does not transfer data, send does
  - unlike SAS which is usually receiver-initiated (load fetches data)
- Communication done at beginning of iteration, so no asynchrony
- Communication in whole rows, not element at a time
- Core similar, but indices/bounds in local rather than global space
- Synchronization through sends and receives
  - Update of global diff and event synch for done condition
  - Could implement locks and barriers with messages
- Can use REDUCE and BROADCAST library calls to simplify code

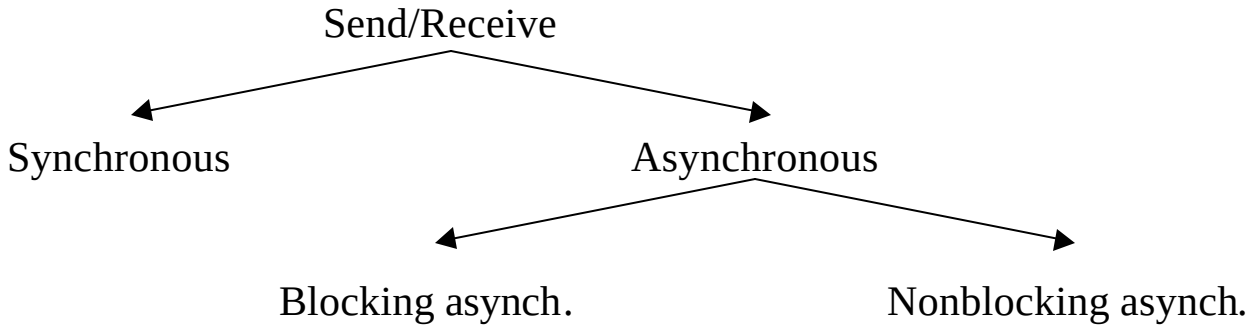
```
      /*communicate local diff values and determine if done, using reduction and broadcast*/
25b.    REDUCE(0,mydiff,sizeof(float),ADD);
25c.    if (pid == 0) then
25i.      if (mydiff/(n*n) < TOL) then done = 1;
25k.    endif
25m.      BROADCAST(0,done,sizeof(int),DONE);
```

Send and Receive Alternatives

Can extend functionality: stride, scatter-gather, groups

Semantic flavors: based on when control is returned

Affect when data structures or buffers can be reused at either end



- Affect event synch (mutual excl. by fiat: only one process touches data)
- Affect ease of programming and performance

Synchronous messages provide built-in synch. through match

- Separate event synchronization needed with asynch. messages

With synch. messages, our code is deadlocked. Fix?

Orchestration: Summary

Shared address space

- Shared and private data explicitly separate
- Communication implicit in access patterns
- No *correctness* need for data distribution
- Synchronization via atomic operations on shared data
- Synchronization explicit and distinct from data communication

Message passing

- Data distribution among local address spaces needed
- No explicit shared structures (implicit in comm. patterns)
- Communication is explicit
- Synchronization implicit in communication (at least in synch. case)
  - mutual exclusion by fiat

Correctness in Grid Solver Program

Decomposition and Assignment similar in SAS and message-passing

Orchestration is different

- Data structures, data access/naming, communication, synchronization

	SAS	Msg-Passing
Explicit global data structure?	**Yes**	**No**
Assignment indept of data layout?	**Yes**	**No**
Communication	**Implicit**	**Explicit**
Synchronization	**Explicit**	**Implicit**
Explicit replication of border rows?	**No**	**Yes**

Requirements for performance are another story ...